

Analysis of Students' Mathematical Reasoning Abilities Using a Question Development Model Based on 21st-Century Skills

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ABSTRACT

Mathematics is a scientific discipline that contributes a lot to human survival. They will get many things, such as developing reasoning, solving problems, and so on, when facing issues related to mathematics, especially in a century that can be said to be super modern, such as the 21st century, which needs cadres with a high level of reasoning. Mathematical reasoning is one of the competencies that students should have at school. This research aims to determine the level of students' mathematical reasoning abilities in mathematics subjects, especially geometry. The method used in this research is qualitative. The technical analysis in this research is qualitative descriptive data analysis. The techniques used in data collection are test questions and interviews. The research subjects were 16 students, each given the same questions. The research results show that (1) Students in the very-good category are 12.5%, (2) Students in the excellent category are 32.5%, (3) Students in the fair category are 32.5%, (4) Students in the poor category are 22.5%. The analysis results show that the maximum gains are felt by students who often use the question development model. It is hoped that teachers will always encourage their students to take the time to solve mathematical problems, especially those related to geometry, so that their mathematical reasoning abilities develop over time.

Keywords:

21st-Century Skills; Capability Analysis; Mathematical Reasoning; Question Development Model; Student Mathematics.

INTRODUCTION

Geometry is a branch of mathematics significant for those who love mathematics because it discusses basic mathematical materials, such as points, lines, planes, and space. It aligns with what Fiantika et al. (2018) stated: geometry is closely related to points, lines, planes, and space. In everyday life, geometry is inseparable from human activities in this world. For example, a construction worker or architect still uses geometric materials to build the building. Geometry may be taught using a problem-creation technique to help students develop their mathematical thinking. Mathematical reasoning is part of complex, high-level mathematical thinking. Therefore, learning that focuses on reasoning abilities requires lower-stage concepts. Students' mathematical reasoning abilities do not exist without good understanding abilities (Bozkuş & Ayvaz, 2018; Indriati, 2018; Misnasanti et al., 2017). With mathematical reasoning, students can examine the advantages and disadvantages of the problems they face to find solutions later. Reasoning is very urgent because mathematics can be understood in one way, using reasoning (Han, 2013; Jack & Thompson, 2017; Mumu & Tanujaya, 2019; Muslimin & Sunardi, 2019).

However, there are a variety of methods for using mathematical reasoning. However, in this article, we will present reasoning utilizing the development of mathematical problems, especially in geometry material. Students are supposed to be able to strengthen their thinking skills through question development, which will help them answer problems successfully in the future. Under what was stated by Rizta et al. (2013), problems solved by reasoning, be it mathematics or others, can be solved well. However, he added that according to current realities, teachers emphasize understanding concepts more

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than higher thought processes, including reasoning. There should be homework for teachers who teach in elementary and middle schools.

Furthermore, according to Rahmawati and Setyaningsih (2019), teachers use lecture or conventional teaching methods. Teachers have not implemented the teaching process through discussions or forming discussion groups, so students do not understand the material presented by the teacher and are less active in participating in the mathematics learning process (Tohir et al., 2020). Students' performance in their mathematical reasoning tests may suffer as a result. Drawing from this perspective, one may argue that educators should utilize appropriate models rather than conventional ones when creating math problems. It aims to ensure that students can develop their reasoning, especially in mathematics material, so they do not appear passive in learning.

However, in mathematical reasoning, several activities must be carried out by students, namely: (1) draw logical conclusions; (2) use explanations using facts, characteristics, models, and relationships; estimate answers and process solutions; (3) use patterns/regularities and relationships to analyze mathematical situations, draw analogies and generalizations; (4) constructing and testing conjectures; (5) provide examples of refutation; follow the rules of inference; (6) checking the validity of the argument; (7) construct valid arguments; (8) compiling direct, indirect proof and using mathematical induction (Anisah, 2013). Math teachers must make an effort to formulate various mathematical problems, particularly in geometry, and to make the issues as good as they can be to foster a fit between themselves and their students, make learning engaging, and give students a broad understanding.

METHOD

The method used in this research is qualitative. Qualitative methods are methods with several characteristics (Mohajan, 2018; Ormston et al., 2014; Priya, 2021; Yilmaz, 2013), including (1) researchers having direct contact with the subjects of their studies, (2) reality is subjective and multiple as seen by participants in the study, and (3) studies in natural situations. A descriptive qualitative technique is employed in this study to describe the outcomes attained by the 16 students in question. The data instruments in this study used observation and test questions and mathematics questions on geometry material. As for the data analysis, it used test questions, observations, and interviews, and it modified the assessment levels under (Azmi, 2013), which is detailed below.

Table 1. Test score level criteria

Score	Criteria	Indicators
4-6	Not Good	Students do not understand the problem satisfactorily or even do not understand the problem they are facing, so they cannot solve it or draw conclusions.
7-10	Pretty Good	Students can understand the problem, but they cannot solve the problem and draw conclusions.
11-13	Good	Students can understand problems, solve them, and conclude reasonably.
14-16	Very Good	Students can understand problems, solve them, and conclude very well and thoroughly.

RESULTS AND DISCUSSION

The students individually generated thirty-two questions after completing the mathematical modeling course. Student explanations of their developed questions occur once a week. The instructor, an authority on mathematical modeling, provided feedback to the students following the talk. Students have generated these questions, and here is the validation step for them. Students develop high-quality mathematical modeling questions that adhere to the qualities of mathematical modeling questions by revising their first drafts in response to feedback and

recommendations. Students studying mathematical modeling have created the two instances of problems that follow. Table 2 displays the test score results for students according to category groups: high, medium, and low.

Table 2. Test results for student category groups

Categories	N	Range	Minimum	Maximum	Sum	Mean	Std. Deviation	Variance
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic
Low	6	25	35	60	271	45.17	4.743	11.618
Medium	5	13	67	80	362	72.40	2.293	5.128
High	5	14	82	96	439	87.80	3.169	7.085

The test results for the participants in the low, medium, and high group categories are displayed in Table 2 and are 45.17, 72.40, and 87.80, respectively. It is evident from these statistics that the three subject groups' scores, 27.23, 15.40, and 27.23, differ. The test's findings indicate that there seems to be a lot of variation in the subject groups' skills. Here is an example of a student-created mathematical modeling issue.

First Modeling Question

The cost of painting a room five meters long, four meters wide, and three meters high is Rp 2,000 per meter. What is the price of painting the room, then?

If there is such a question, students will vary in their solutions.

a. Students in the low-good category

Because the room in issue is formed like a block, it is likely that students in the poor group may not comprehend the problem and may not even be aware of the formula for blocks. According to Tri Roro Suprihatin (2018), a teacher-centered learning approach is partially to blame for this group of students' passive demeanor, which suggests that there is only one way to learn. It is also consistent with Sulistiawati's (2014) assertion that one of the causes of students' inferior mathematical reasoning ability is non-student-centered mathematics learning. According to the existing reality, many students are still in that category in the classroom. The following are the results of the researcher's interview with one of the students in this category.

Researcher : How is your work?
 Student : I haven't found the answer yet, sir.
 Researcher : How come?
 Student : I don't understand the problem.
 Researcher : That's the problem of finding the surface area of a disk block.
 Student : Oh yes, sir, and you also don't know the formula for the surface area of a block.
 Researcher : Oh, next time, study harder, sir; if possible, look for a private teacher so you can understand better.
 Student : Yes, sir, thank you.
 Researcher : You're welcome.

b. Students in the pretty-good category

Students in the pretty-good category may be able to understand what is meant by the problem above. Their response can be inaccurate because it only applies to pupils who are aware of the circumstances. At the same time, there is still a little confusion about whether to use the

formula for the volume of a block or its surface area for the solution. The following are the results of students' work in the sufficient category on the above problems.

c. Students in the good category

Students in the good category will understand the problem in question well, and they can answer it in several steps and draw conclusions well, too. Students in this category include the standards students expect when learning takes place. According to the good category students, the following is the solution to the problem above.

d. Students in the very-good category

Students in the very-good category will understand the related problems well by using the suitable model and finding solutions. The conclusions drawn are very good. He will add information based on the issue. According to Sumartini (2015), this is partly caused by students often using mathematical models for problem-solving. The more he uses this model, his mathematical reasoning will improve. The following are the results of the questions above from students in the very-good category.

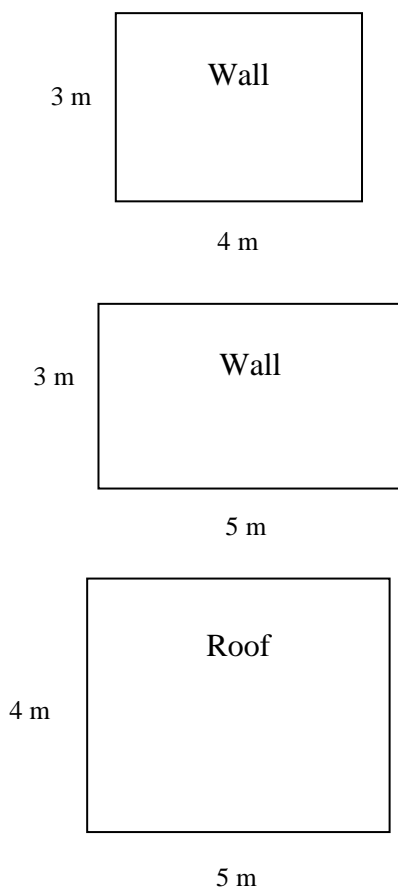
1. Good Grade Category Work Results

Known : -) $p = 5, l = 4, t = 3$

-) Paint cost per meter = Rp. 2000

Asked : What is the total cost...?

Answer : Two pairs of walls and one roof area are to be painted. The floor is not painted.
Here is the picture.



$$\begin{aligned} L^1 &= p \times l \\ &= 4 \times 3 \\ &= 12 \times 2 = 24 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} L^2 &= p \times l \\ &= 5 \times 3 \\ &= 15 \times 2 = 30 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} L^3 &= p \times l \\ &= 5 \times 4 \\ &= 20 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Total cost} &= (24 + 30 + 20) \times 2000 \\ &= 74 \times 2000 \\ &= 148000 \end{aligned}$$

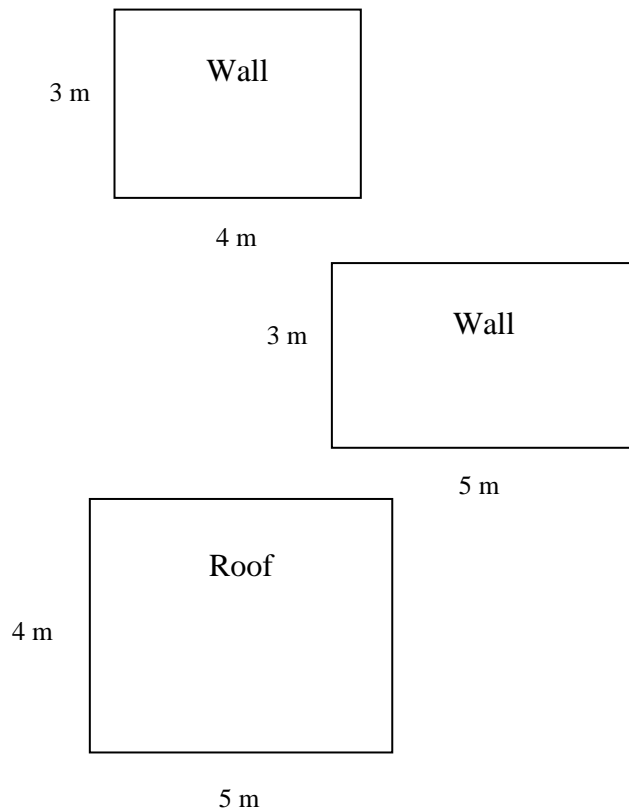
This cost is without using ventilation and windows. If you use both, then the total cost >148000

2. Medium Value Category Work Results

Known : -) $p = 5, l = 4, t = 3$
 -) Paint cost per meter = Rp. 2000

Asked : What is the total cost...?

Answer : Two sets of walls and one section of the roof need painting. There is no paint on the floor. Here is the picture.



$$\begin{aligned} L^1 &= p \times l \\ &= 4 \times 3 \\ &= 12 \times 2 = 24 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} L^2 &= p \times l \\ &= 5 \times 3 \\ &= 15 \times 2 = 30 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} L^3 &= p \times l \\ &= 5 \times 4 \\ &= 20 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Total cost} &= (24 + 30 + 20) \times 2000 \\ &= 74 \times 2000 \\ &= 148000 \end{aligned}$$

Students in this category answered correctly but responded to the question incompletely.

3. Low-Value Category Work Results

Students who fall into this category do not correctly explain the answers to the questions. In other words, this category of students answered directly, unlike the two categories above. Here are the answers.

$$\begin{aligned} \text{Block surface area} &= (p \times l).(2) + (l \times t).(2) + (p \times t) \\ &= (5 \times 4) + (4 \times 3).(2) + (5 \times 3).(2) \\ &= 20 + 24 + 30 \\ &= 74 \times 2000 \\ &= 148000 \end{aligned}$$

Some students get the minimum score if their answers are as follows.

$$\begin{aligned} \text{Block surface area} &= (p \times l) + (l \times t) + (p \times t) \\ &= (5 \times 4) + (4 \times 3) + (5 \times 3) \\ &= 20 + 12 + 15 \\ &= 47 \times 2000 \\ &= 94000 \end{aligned}$$

Even though the formula they used was correct, they did not realize that there were two pairs of walls in question, so the answer they produced was wrong.

The following are the SPSS results of the scores obtained by the 16 students who were the research objects.

Second Modeling Question

Do you agree that circles have no sides?

A. The student's answer is in the good grades category

Researcher : What do you think about circles? Does a circle have sides or not?

Student : A circle is a flat shape that has unlimited sides

Researcher : Why do you think that?

Student : It is infinite; thus, you may rotate the corner points indefinitely.

B. student answers in the medium value category

Researcher : What do you think about circles? Does a circle have sides or not?

Student : A circle is a flat shape that has unlimited sides.

Researcher : Why do you think that?

Student : Because it is played to the maximum extent.

C. Students' answers are in the low score category

Researcher : What do you think about circles? Does a circle have sides or not?

Student : A circle is a flat shape that has no sides

Researcher : Why do you think that?

Student : That's because there are no corner points.

The results above show that students lack attention to developing knowledge by solving problems. It causes their lack of interest in learning mathematics, and it is the task of teachers to motivate their students to learn by developing questions. The wording below indicates whether or not the results are typical.

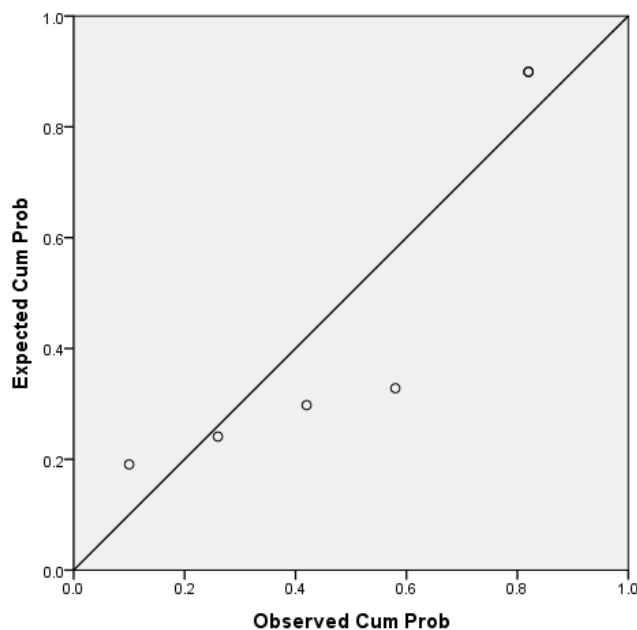


Figure 1. P-Plot Results Low Value

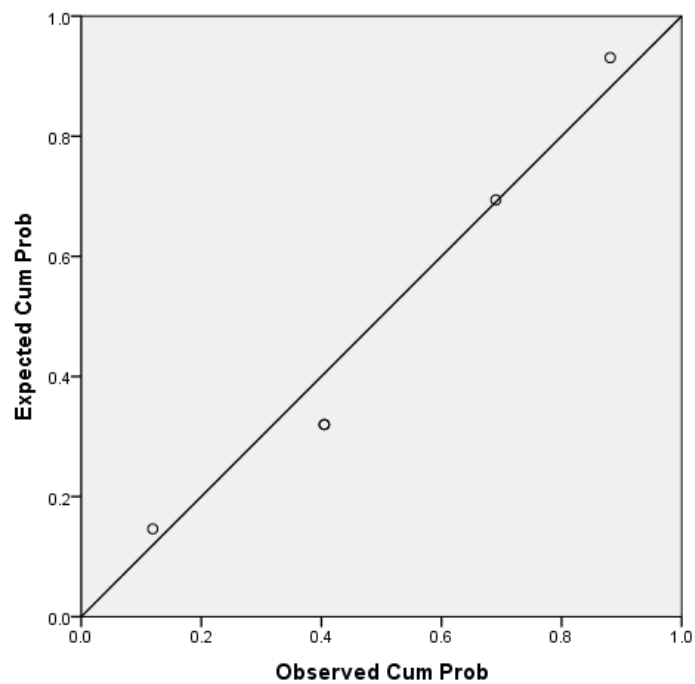


Figure 2. Medium Value P-Plot Results

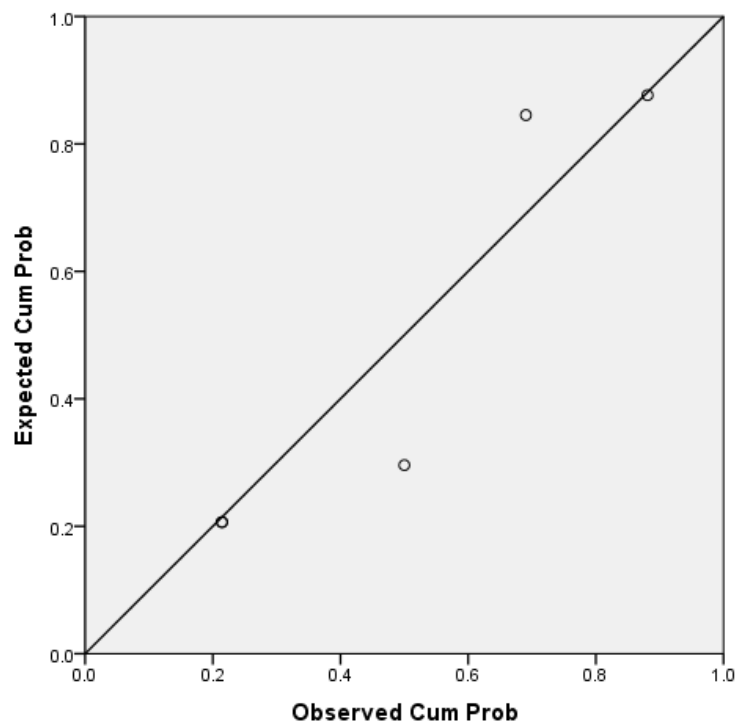


Figure 3. High-Value P-Plot Results

The research can be declared normal based on the results above because the small circles in the SPSS image results are not far from a straight line. The following graph may be created by examining the answers to the questions provided by 16 students.

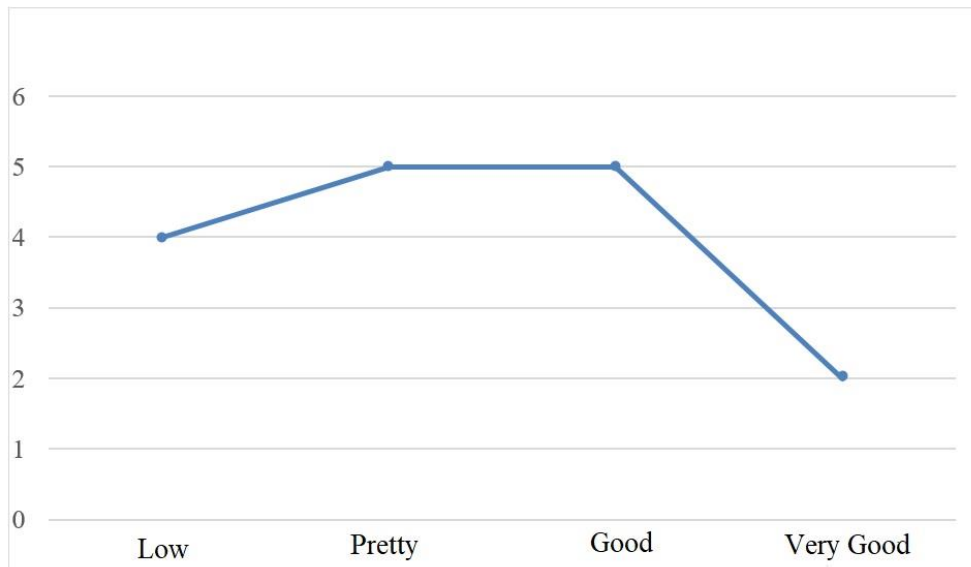


Figure 3. Graph of student answers based on category groups

The graph above shows that the 16 students who were the research target did not all get good grades. As can be observed, four individuals received low marks (scoring 4-6), five people got a fair score (score 7-10), five people got a good score or category (score 11-13), and two people got a reasonably satisfactory score, namely the very-good category (14-16). A teacher must thus take the initiative to think up questions frequently. The purpose of these well-crafted, complex questions is to help students hone their thinking abilities. Apart from that, there must also be supervision from educators so that students can be helped a little by their presence when they have difficulties with the problems they face. One of the mathematical topics covered in class is linear programming (Kenney et al., 2020; Neogy et al., 2018; Sole, 2016). When students make mistakes or struggle to convert a problem into a mathematical model, they are solving issues unrelated to linear programming, which can lead to errors in problem-solving. In making graphs, students are confused in determining the solution area on the graph, and students have difficulty determining the coordinate points and intersection points on the graph (Bollen et al., 2017; Boote, 2014; Brown & Hurst, 2012; Ivanjek et al., 2016; Martínez-Planell & Trigueros Gaisman, 2012).

CONCLUSION

Based on the results of the research and discussion it showed that that there are still many students who are not able to solve the questions well. Even more, they don't understand the problems they face well, so there are no solutions for the issues they encounter. The question development model helps with that. On the other hand, it can also develop students' mathematical reasoning. With the question development model, students will be given questions step by step, in the sense that they will get questions from low to high.

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