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Utilizing the Hybrid Models of Double Exponential Smoothing and Double Moving Average with Fuzzy Time Series Markov Chain for Stock Price Forecasting

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ABSTRACT

Double exponential smoothing (DES) and double moving average (DMA) are forecasting algorithms that are well-suited for time series data with trend patterns as they can be used to quickly identify the trend direction by smoothing the data. However, both models tend to give late signals and can only capture linear relationships in the data. Therefore, they should be combined with other models that are effective in nonlinear modeling, such as fuzzy time series Markov chain (FTSMC), for forecasting financial time series data, such as stock prices, that have not only linear relationships but also nonlinear relationships. This paper proposed the use of hybrid approaches of DES-FTSMC and DMA-FTSMC models to analyze the trend time series data. The models are combined in order to effectively capture different forms of patterns in the data. The hybrid models utilize DES and DMA to identify the direction of the trend and FTSMC to model the residual series after the removal of the trend effect. The proposed hybrid approaches are applied to the daily closing stock price of PT Indosat Ooredoo Hutchison Tbk. The results show that the DES-FTSMC hybrid model generates a MAPE value of 1.09% on the training data and 0.89% on the testing data. While the DMA-FTSMC combination yields MAPE values of 1.24% and 1.19% on the training and testing data respectively. This finding suggests that the proposed hybrid DES-FTSMC model is the better forecasting model for achieving higher accuracy.

Keywords:

hybrid approach; double exponential smoothing; double moving average; fuzzy time series Markov chain; stock price

INTRODUCTION

Stocks are one of the most popular investment instruments among investors due to their significant profit potential. However, the movement of stock prices is influenced by a multitude of complex factors. Consequently, despite the potential for high returns, investing in stocks comes with equally high risks. A forecasting approach can be a useful tool in helping to determine stock price movements and prospects as it allows investors to plan and make investment decisions in order to reduce the risk of potential losses.

The forecasting of trend-patterned time series data can be obtained through the application of double exponential smoothing (DES) and double moving average (DMA) models, according to the findings of [1], who assert that DES and DMA represent stochastic trend methods that develop local linear trends in time series data, thereby enabling the provision of accurate forecasting results. Previous studies have utilized these two models to perform forecasting on trend-patterned time series data. Research by [2] and [3] shows that the DES and DMA models are able to forecast data with high accuracy. However, the two models have a significant drawback in that they tend to give late signals and can only capture linear relationships in the data. Concurrently, according to [4], fluctuating changes in stock prices allow stock price data to not only show linear relationships but also contain nonlinear relationships. Therefore, a forecasting approach that can capture both linear and nonlinear relationships simultaneously is required.

Inspired by [5], numerous hybrid approaches have been proposed by researchers, combining two or more models, with the aim of addressing linear and nonlinear time series problems in a unified manner, while preserving the existing data patterns and improving the forecasting accuracy by optimizing the benefits of each component of the model. In a recent study, [6] discussed a hybrid model based on the triple exponential smoothing and fuzzy time series Markov chain (FTSMC) to improve the forecasting accuracy of trend and seasonal data. Following the previous research, [7] proposed a hybrid approach by combining autoregressive integrated moving averages (ARIMA) and FTSMC to enhance the forecasting performance of long-memory trend data. Recently, [8] also conducted research using a hybrid method that combines the simple exponential smoothing (SES) with the fuzzy time series (FTS) model.

This paper focuses on DES-FTSMC and DMA-FTSMC hybrid approaches in order to improve the forecasting accuracy of trend-patterned time series data. The proposed hybrid approaches are then implemented to the daily closing stock price of PT Indosat Ooredoo Tbk data. The results of the forecasting presented in the illustrated examples show that the proposed DES-FTSMC hybrid model gives better performance than the hybrid model of DMA-FTSMC.

METHOD

A brief overview of the DES, DMA, FTSMC, and the hybrid methods proposed is provided herewith.

Double Exponential Smoothing (DES)

Exponential smoothing is a forecasting method that assigns greater weight to historical data as it accumulates over time. This weighting is exponential, meaning that the smoothing method continuously improves forecasts for the latest forecasting objects by giving exponentially decreasing priority to older observation objects [9]. The DES method employs two distinct smoothing constants, designated α (level smoothing constant) and β (trend smoothing constant), with values ranging from 0 to 1, to directly smooth levels and trends. As outlined in [1], the steps involved in the DES method include the calculation of the level estimate smoothing value and the trend estimate smoothing value, which are determined by the following equations.

$$L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + T_{t-1}) \tag{1}$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1} \tag{2}$$

This is then followed by the calculation of the forecasting value, which is derived from the following equation.

$$\hat{Y}_{t+p} = L_t + pT_t \tag{3}$$

The initial values for the level estimation and trend estimation smoothing values in period $t = 1$ are determined according to the following equation [10].

$$L_1 = Y_1 \tag{4}$$

$$T_1 = \frac{(Y_2 - Y_1) + (Y_4 - Y_3)}{2} \tag{5}$$

Double Moving Average (DMA)

The moving average method involves collecting a set of observations and calculating their average. The aforementioned average is then used as a forecast estimate for the next period. The fundamental basis of the DMA method is to calculate the moving average of the first moving average, which is used to overcome certain inherent limitations of the SMA method [9]. According to [1], the steps involved in the DES method include the calculation of the first set moving average and the second set moving average with the following equations.

$$S'_t = \frac{Y_t + Y_{t-1} + Y_{t-2} + \cdots + Y_{t-n-1}}{n} \tag{6}$$

$$S''_t = \frac{S'_t + S'_{t-1} + S'_{t-2} + \cdots + S'_{t-n-1}}{n} \tag{7}$$

Subsequently, the constant value and trend coefficient are calculated in accordance with the equation below.

$$a_t = 2S'_t - S''_t \tag{8}$$

$$b_t = \frac{2}{n-1}\left(S_t' - S_t''\right) \quad (9)$$

The forecasting value based on the DMA model can be calculated using the following equation.

$$\hat{Y}_{t+p} = a_t + b_t p \quad (10)$$

Fuzzy Time Series Markov Chain (FTSMC)

The FTS approach was initially proposed by [11]. This model is capable of working in a limited number of data sets and such data sets do not necessitate the assumption of linearity. This paper applies the FTSMC, a new concept first proposed by [12] in his research on the analysis of the accuracy of Taiwan currency exchange rate prediction with the US dollar. The algorithm proposed by [12] is presented in the following steps.

Step 1: Set the universe of discourse U, which is defined as U = [D_{min} – D_1, D_{max} + D_2] and divide into several equal parts, u_1, u_2, …, u_n. Dmin and Dmax, respectively, represent the minimum and maximum values observed in the known historical data, while D_1 and D_2 are two proper positive numbers determined by the researcher.

Step 2: Divide the universe of discourse U into equal-length intervals.

Step 3: Determine fuzzy sets A_1, A_2, …, A_n on the universe of discourse U and fuzzify the historical data. Each A_i is established by the intervals u_1, u_2, …, u_n.

Step 4: Define fuzzy logical relationships (FLRs)

Step 5: Obtain the fuzzy logical relationship groups (FLRGs) by taking together fuzzy propositions that share the same antecedent.

Step 6: Determine the Markov transition probability matrix

Step 7: Obtain the initial forecasting value. The calculations are conducted by the following rules:
Case 1: If the fuzzified value of the data at time t - 1 is A_i and the FLRG of A_i is empty (i.e., $A_i \rightarrow \emptyset$), with the maximum membership value of A_i occurring in interval u_i, then the forecasting value of time t is the midpoint of interval u_i, namely m_i.
Case 2: If the fuzzified value of the data at time t - 1 is A_i and based on the formed FLRG A_i has a one-to-one relationship, such as $A_i \rightarrow A_j$ where $P_{i,j} = 1$ and $P_{i,k} = 0$, $j \neq k$, then the forecasting value for time t is m_j, where m_j is the midpoint of interval u_j.
Case 3: If the fuzzified value of the data at time t - 1 is A_i and based on the formed FLRG A_i has a one-to-many relationship, such as $A_i \rightarrow A_1, A_2, \ldots, A_n$ with j = 1, 2, …, n, then the forecasting value of time t can be solved by the equation

$$\hat{Y}_t = m_1 P_{j1} + \cdots + m_{j-1} P_{j(j-1)} + Y_{t-1} P_{jj} + m_{j+1} P_{j(j+1)} + \cdots + m_n P_{jn} \quad (11)$$

where m_1, m_2, …, m_{j-1}, m_{j+1}, …, m_n is the midpoint of u_1, u_2, …, u_{j-1}, u_{j+1}, …, un and the value of m_j is replaced with Y_{t-1} to retrieve more information from state A_j in time t - 1.

Step 8: Adjust the tendency of the forecasting values using the following rules
Case 1: If state A_i communicates with A_i and makes an increasing transition into state A_j at time t, (i < j), then the adjusting value Dt is defined as Dt = $l/2$ where l is the length of the class interval.
Case 2: If state A_i communicates with A_i and makes a decreasing transition into state A_j at time t, (i < j), then the adjusting value Dt is defined as Dt = -($l/2$), where l is the length of the class interval.
Case 3: If state A_i makes a jump-forward transition into state A_{i+v} at time t, ($1 \leq v \leq n-1$), then the adjusting value Dt is defined as Dt = $(l/2)v$, where l is the length of the class interval and v is the number of leaps forward.
Case 4: If state A_i makes a jump-backward transition into state A_{i-v} at time t, ($1 \leq v \leq n$), then the adjusting value Dt is defined as Dt = $-(l/2)v$, where l is the length of the class interval and v is the number of leaps backward.

Step 9: Obtain the adjusted forecasting result of time t. The adjusted forecasting value can be obtained by combining the initial forecasting value described in step 7 and the forecasting adjustment value described in step 8.

Hybrid Model

The use of hybrid models that combine two or more forecasting methods was first introduced by [5], who stated that it is necessary to analyze both linear and nonlinear relationships simultaneously in order to solve time series cases. In addition, it is known that the DES and DMA models can be utilized to ascertain the direction of the trend as both models can smooth the action of data and filter out the noise. In contrast, FTSMC is effective for the analysis of stationary nonlinear time series data.

In this paper, DES and DMA will be utilized as the first components of the proposed hybrid approaches that capture particular trend components. The residuals from DES and DMA models will become stationary series and still contain information about the nonlinear relationship. Therefore, they must be modeled by FTSMC as the second component. The residuals, e_t, can be calculated by

$$e_t = Y_t + \hat{Y}_{1t} \tag{12}$$

where Y_t is the actual value for time t and $\hat{Y}_{1t}$ is the forecast value from either DES or DMA model for time t.

The proposed hybrid models are constructed in two steps. The process begins with modeling the trend part using the DES and DMA models. This is followed by modeling the residuals from DES and DMA models using the FTSMC model. The results of the FTSMC model may be regarded as the forecast of the error term for the DES and DMA models. Thus, the forecast of the time series by the proposed hybrid approaches can be regarded as

$$\hat{Y}_t = \hat{Y}_{1t} + \hat{Y}_{2t} \tag{13}$$

where $\hat{Y}_{1t}$ is the forecast value at time t obtained by the first component of the proposed hybrid approach, which in this case is either DES or DMA, and $\hat{Y}_{2t}$ is the forecast value of the residuals of DES or DMA models at time t obtained by FTSMC model.

Forecasting Accuracy Evaluation

The mean absolute percentage error (MAPE) value will be used to evaluate the forecasting accuracy of all models. As stated by [13], a model is considered good if its MAPE value is below 20%. The smaller the value, the more effective the model's forecasting abilities. The MAPE value can be calculated by first determining the absolute error for each period and then dividing it by the actual observed value for that period. The resulting value is then averaged to obtain the MAPE. The equation for calculating the MAPE value is as follows.

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\frac{|Y_t - \hat{Y}_t|}{|Y_t|} \times 100\% \tag{14}$$

RESULTS AND DISCUSSION

The proposed hybrid models are applied to the daily closing stock price of PT Indosat Ooredoo Hutchison Tbk. The research data comprises the daily closing stock price data of PT Indosat Ooredoo Hutchison Tbk for the period from January 2, 2023, to December 29, 2023, with a total of 239 data points. The data set was divided into two distinct subsets, training data and testing data. The training data set constituted 80% of the total data set, while the testing data set comprised the remaining 20%. This resulted in a total of 192 training data points and 47 testing data points. The time series plot of the data illustrates the fluctuating closing stock prices of PT Indosat Ooredoo Hutchison Tbk over time. This indicates that the data does have a trend data pattern (see Figure. 1).

DES-FTSMC Hybrid Model

The DES-FTSMC hybrid model involves the application of the DES model to address the trend patterns in the data, followed by the utilization of the residuals obtained from the DES modeling process in the subsequent FTSMC modeling. The results of both methods are combined using a hybrid formula. The steps to construct the proposed DES-FTSMC hybrid model for the data are given below.

Step 1: Determine the values of the level smoothing constant and trend smoothing constant, in this case α = 0.789 and β = 0.035 are chosen, then apply (1) and (2) to obtain the level estimated smoothing value and trend estimated smoothing value. Next, apply (3) to achieve the first forecast component, $\hat{Y}_{1t}$, and the residuals, e_t. The results of the analysis using the DES method for training data can be found in Table 1.

Step 2: Model the residuals obtained from the DES model using the FTSMC model and get the second forecast component, $\hat{Y}_{2t}$. In this instance, 200 is specified as the length of the interval. Forecast value for the residuals can be obtained through the following steps.

(1) Set the universe of discourse U for the residuals. From the data obtained the minimum residual, D_{min}, is -757,3758 and the maximum residual, D_{max}, is 484,2793 with D_1 = 142,6242 and D_2 = 215,7207. Therefore, U = [-900, 700].
(2) Determine fuzzy sets $A_1, A_2, \ldots, A_k$ on the universe of discourse U and fuzzify the historical data. U is divided into 8 intervals i.e. u_1 = [-900, -700], u_2 = [-700, -500], …, u_8 = [500, 700].
(3) Define the FLRs and FLRGs.
(4) Calculate the forecast output.

The forecasting results can be found in Table 2.

Step 3: Obtain the final forecast values by adding $\hat{Y}_{1t}$ and $\hat{Y}_{2t}$ as (12). The final forecast value for the hybrid model DES-FTSMC for training data can be found in Table 3.

Step 4: Apply steps 1, 2, and 3 on the testing data using the model that has been derived from the training data. The final forecast value for the hybrid model DES-FTSMC for testing data can be found in Table 4.

Step 5: Apply (14) to obtain the MAPE value of the forecasting results. The results of forecasting with the DES-FTSMC hybrid model yielded a MAPE value of 1.09% on the training data and 0.89% on the testing data.

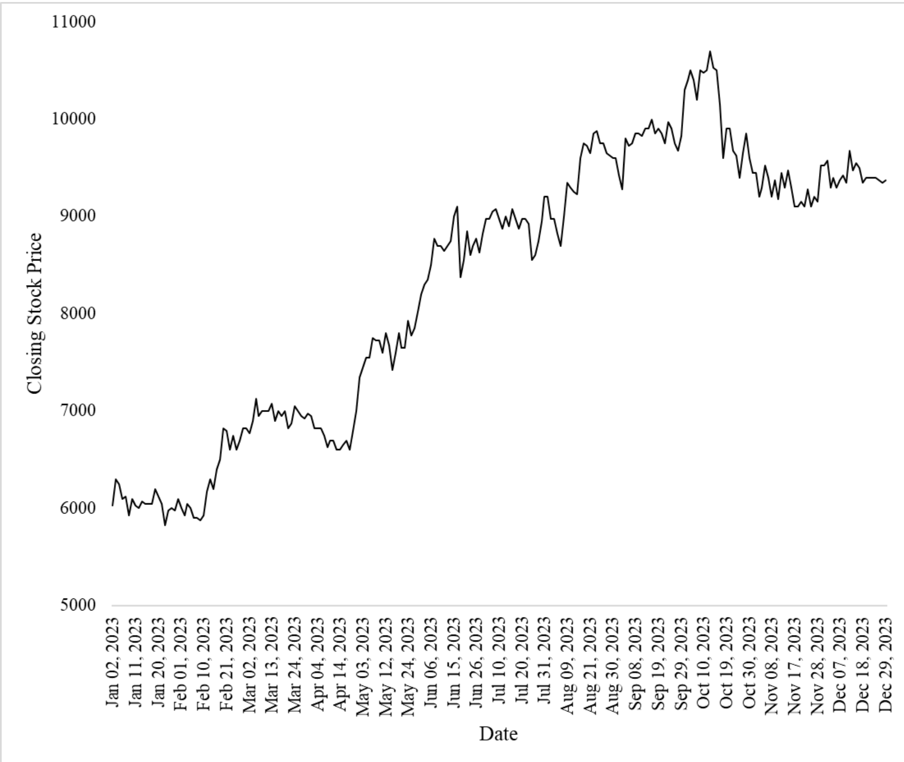

Figure 1. Time series plot of daily closing stock prices of PT Indosat Ooredoo Hutchison Tbk for the period January 2, 2023 - December 29, 2023.

Table 1. The results of the analysis using the DES method for training data

t	Y_t	L_t	T_t	$\hat{Y}_{1t}$	$e_t = Y_t - \hat{Y}_{1t}$
1	6025	6025,000	62,500	-	-
2	6300	6255,270	68,426	6087,500	212,500
3	6250	6265,513	66,371	6323,696	-73,696
4	6100	6148,810	59,904	6331,883	-231,883
5	6125	6142,622	57,570	6208,715	-83,715
…	…	…	…	…	…
189	9600	9736,963	-1,159	10250,670	-650,670
190	9900	9865,438	3,420	9735,805	164,195
191	9900	9893,445	4,289	9868,858	31,142
192	9675	9721,884	-1,923	9897,733	-222,733

Table 2. The results of the calculation of the fitted value of the DES model residue using FTSMC for the training data

t	e_t	Fuzzification	$\hat{e}_t$	D_t	$\hat{Y}_{2t} = \hat{e}_t + D_t$
1	-	-	-	-	-
2	212,500	A_6	-	-	-
3	-73,696	A_5	41,214	-100	-58,786
4	-231,883	A_4	-80,666	-100	-180,666
5	-83,715	A_5	-5,932	100	94,068
…	…	…	…	…	…
189	-650,670	A_2	-300,000	-100	-400,000
190	164,195	A_6	0,000	400	400,000
191	31,142	A_5	28,159	-100	-71,841
192	-222,733	A_4	-23,968	-100	-123,968

Table 3. The final forecast value of the hybrid model DES-FTSMC for training data

t	Y_t	$\hat{Y}_{1t}$	$\hat{Y}_{2t}$	$\hat{Y}_t$	$\frac{\lvert Y_t - \hat{Y}_t \rvert}{Y_t}$
1	6025	-	-	-	-
2	6300	6087,500	-	-	-
3	6250	6323,696	-58,786	6264,910	0,24%
4	6100	6331,883	-180,666	6151,217	0,84%
5	6125	6208,715	94,068	6302,783	2,90%
…	…	…	…	…	…
189	9600	10250,670	-400,000	9850,670	2,61%
190	9900	9735,805	400,000	10135,805	2,38%
191	9900	9868,858	-71,841	9797,017	1,04%
192	9675	9897,733	-123,968	9773,766	1,02%
		MAPE			1,09%

Table 4. The final forecast value of the hybrid model DES-FTSMC for testing data

t	Y_t	$\hat{Y}_{1t}$	$\hat{Y}_{2t}$	$\hat{Y}_t$	$\frac{\lvert Y_t - \hat{Y}_t \rvert}{Y_t}$

193	9625	9719,962	95,7705	9815,732	1,98%
194	9400	9640,418	-192,17	9448,251	0,51%
195	9650	9439,332	192,48	9631,812	0,19%
196	9850	9600,255	40,7193	9640,974	2,12%
197	9600	9798,994	-148,72	9650,274	0,52%
…	…	…	…	…	…
235	9400	9396,004	-37,099	9358,905	0,44%
236	9400	9396,719	-38,649	9358,070	0,45%
237	9375	9396,960	-39,035	9357,925	0,18%
238	9350	9376,661	-52,687	9323,975	0,28%
239	9375	9351,907	-55,229	9296,678	0,84%
		MAPE			0,89%

Table 5. The results of the analysis using the DMA method for training data

t	Y_t	S'_t	S''_t	a_t	b_t	$\hat{Y}_{1t}$	$e_t = Y_t - \hat{Y}_{1t}$
1	6025	-	-	-	-	-	-
2	6300	-	-	-	-	-	-
3	6250	6191,67	-	-	-	-	-
4	6100	6216,67	-	-	-	-	-
5	6125	6158,33	6188,89	6127,78	-30,56	-	-
6	5925	6050,00	6141,67	5958,33	-91,67	6097,22	-172,222
7	6100	6050,00	6086,11	6013,89	-36,11	5866,67	233,333
8	6025	6016,67	6038,89	5994,44	-22,22	5977,78	47,222
9	6000	6041,67	6036,11	6047,22	5,56	5972,22	27,778
10	6075	6033,33	6030,56	6036,11	2,78	6052,78	22,222
…	…	…	…	…	…	…	…
188	10150	10391,67	10513,89	10269,44	-122,22	10586,11	-436,111
189	9600	10083,33	10350,00	9816,67	-266,67	10147,22	-547,222
190	9900	9883,33	10119,44	9647,22	-236,11	9550,00	350,000
191	9900	9800,00	9922,22	9677,78	-122,22	9411,11	488,889
192	9675	9825,00	9836,11	9813,89	-11,11	9555,56	119,444

Table 6. The results of the calculation of the fitted value of the DMA model residue using FTSMC for the training data

t	e_t	Fuzzification	$\hat{e}_t$	D_t	$\hat{Y}_{2t} = \hat{e}_t + D_t$
1	-	-	-	-	-
2	-	-	-	-	-
3	-	-	-	-	-
4	-	-	-	-	-
5	-	-	-	-	-
6	-172,222	A_4	-	-	-
7	233,333	A_6	-46,862	200	153,138
8	47,222	A_5	24,761	-100	-75,239
9	27,778	A_5	-2,569	0	-2,569
10	22,222	A_5	-11,806	0	-11,806

…	…	…	…	…	…
188	-436,111	A_3	-36,234	-100	-136,234
189	-547,222	A_2	-62,305	-100	-162,305
190	350,000	A_7	0,000	500	500,000
191	488,889	A_7	238,887	0	238,887
192	119,444	A_6	254,318	-100	154,318

Table 7. The final forecast value of the hybrid model DMA-FTSMC for training data

t	Y_t	$\hat{Y}_{1t}$	$\hat{Y}_{2t}$	$\hat{Y}_t$	$\frac{\lvert Y_t - \hat{Y}_t \rvert}{Y_t}$
1	6025	-	-	-	-
2	6300	-	-	-	-
3	6250	-	-	-	-
4	6100	-	-	-	-
5	6125	-	-	-	-
6	5925	6097,222	-	-	-
7	6100	5866,667	153,138	6019,805	1,31%
8	6025	5977,778	-75,239	5902,539	2,03%
9	6000	5972,222	-2,569	5969,653	0,51%
10	6075	6052,778	-11,806	6040,972	0,56%
…	…	…	…	…	…
188	10150	10586,111	-136,234	10449,877	2,95%
189	9600	10147,222	-162,305	9984,917	4,01%
190	9900	9550,000	500,000	10050,000	1,52%
191	9900	9411,111	238,887	9649,998	2,53%
192	9675	9555,556	154,318	9709,874	0,36%
		MAPE			1,24%

Table 8. The final forecast value of the hybrid model DMA-FTSMC for testing data

t	Y_t	$\hat{Y}_{1t}$	$\hat{Y}_{2t}$	$\hat{Y}_t$	$\frac{\lvert Y_t - \hat{Y}_t \rvert}{Y_t}$
193	9625	9802,778	-201,271	9601,507	0,24%
194	9400	9627,778	-48,794	9578,983	1,90%
195	9650	9283,333	233,814	9517,147	1,38%
196	9850	9436,111	240,738	9676,849	1,76%
197	9600	9727,778	-54,015	9673,763	0,77%
…	…	…	…	…	…
235	9400	9305,556	11,944	9317,500	0,88%
236	9400	9400,000	19,861	9419,861	0,21%
237	9375	9411,111	-25,000	9386,111	0,12%
238	9350	9380,556	-42,153	9338,403	0,12%
239	9375	9347,222	-39,514	9307,708	0,72%
		MAPE			1,19%

Table 9. Comparison of the forecast result for the hybrid models DES-FTSMC and DMA-FTSMC

Hybrid Models	Data	MAPE
DES-FTSMC	Training	1.09%
	Testing	0.89%
DMA-FTSMC	Training	1.24%
	Testing	1.19%

DMA-FTSMC Hybrid Model

The DMA-FTSMC hybrid model can be determined in the same way by addressing the trend patterns in the data using the DMA model, followed by the utilization of the residuals obtained from the DMA modeling process in the subsequent FTSMC modeling. The results of both methods are combined using a hybrid formula. The steps to construct the proposed DMA-FTSMC hybrid model for the data are given below.

Step 1: Determine the time order to calculate the average, in this case a 3x3 time order is chosen, then apply (6) and (7) to get the first set moving average and second set moving average values. Followed by applying (8) and (9) to get the constant value and trend coefficient. Subsequently, apply (10) to obtain the first component, $\hat{Y}_{1t}$, and the residuals, e_t. The results of the analysis using the DMA method for training data can be found in Table 5.

Step 2: Model the residuals obtained from the DMA model using the FTSMC model and obtain the second forecast component, $\hat{Y}_{2t}$. In this instance, 200 is specified as the length of the interval. Forecast value for the residuals can be obtained through the following steps.

(1) Set the universe of discourse U for the residual historical data. From the data obtained the minimum residual, D_{min}, is -830,5556 and the maximum residual, D_{max}, is 650 with D_1 = 69,444 and D_2 = 50. Therefore, U = [-900, 700].
(2) Determine fuzzy sets A_1, A_2, …, A_k on the universe of discourse U and fuzzify the historical data. U is divided into 8 intervals i.e. u_1 = [-900, -700], u_2 = [-700, -500], …, u_8 = [500, 700].
(3) Define the FLRs and FLRGs.
(4) Calculate the forecast output.

The forecasting results can be found in Table 6.

Step 3: Obtain the final forecast values by adding $\hat{Y}_{1t}$ and $\hat{Y}_{2t}$ as (12). The final forecast value for the hybrid model DMA-FTSMC for training data can be found in Table 7.

Step 4: Apply steps 1, 2, and 3 on the testing data using the model that has been derived from the training data. The final forecast value for the hybrid model DMA-FTSMC for testing data can be found in Table 8.

Step 5: Apply (14) to obtain the MAPE value of the forecasting results. The results of forecasting with the DMA-FTSMC hybrid model yielded a MAPE value of 1.24% on the training data and 1.19% on the testing data.

The comparison forecast results of the DES-FTSMC and DMA-FTSMC are listed in Table 9. The results show that the proposed hybrid approach of DES-FTSMC yields smaller MAPE, both in forecasting training and testing data, than the DMA-FTSMC model.

CONCLUSION

The development of more efficient and effective time series forecasting models has has always been a significant challenge, given the crucial importance of forecasting accuracy, especially for investors to make investment decisions in order to reduce the risk of potential losses. By considering the advantages and disadvantages of each component of the hybrid model, it becomes evident that combining two or more different models will enhance the forecasting accuracy. This research study proposes hybrid approaches combining DES and DMA with FTSMC. The process of hybridizing the models commences with either DES or DMA, followed by FTSMC to model the residuals from the first model. The research conducted with the PT Indosat Ooredoo Hutchison Tbk closing stock price data shows that the proposed hybrid approaches utilize the DES and DMA models to remove the trend pattern from the data and

FTSMC to model the residuals obtained from the either DES or DMA models and the define the adjusted forecasting values to enhance the performance of forecasting models. The forecasting results of the DES-FTSMC hybrid model yielded a MAPE value of 1.09% on the training data and 0.89% on the testing data. In comparison, the DMA-FTSMC combination yielded MAPE values of 1.24% and 1.19% on the training and testing data respectively. The findings from the illustrated example suggest that the proposed hybrid model of DES-FTSMC is able to produce better forecasts than the DMA-FTSMC hybrid model, as indicated by its smaller MAPE value in forecasting both the training and testing data.

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