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Undergraduate students' errors in integral calculus: A cognitive load theory perspective

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Abstract:
This study aims to analyze students' errors in solving integral calculus problems based on Newmann's Error Analysis (NEA) and relate them to the Cognitive Load Theory (CLT) perspective. This study employs a descriptive quantitative approach, involving 74 students selected by purposive sampling because they have learned integral material in calculus courses. Data were obtained through an integral calculus test and analyzed using descriptive statistics, including grouping errors according to NEA, tabulating frequencies and percentages, calculating proportions and standard deviations of proportions, and presenting data in tables. The results showed that the most dominant student errors were transformation errors with a high category, which are related to difficulties in choosing the right integral technique, followed by process skill errors due to inappropriate algebraic procedures or symbol manipulation, and comprehension errors, which indicate a relatively good understanding of the problem concept, both in the moderate category. Reading errors and encoding errors were not found, indicating that students were able to read and write answers correctly. From a CLT perspective, the high intrinsic cognitive load in integral calculus material causes transformation and process skill errors, while process skill errors also reflect extraneous cognitive load due to inefficient information management. Moderate comprehension errors indicate that germane cognitive load is managed quite well, although students still have difficulty selecting the appropriate technique and performing algebraic manipulations. These results confirm that students' primary difficulties in integral calculus lie in selecting integration techniques and processing information during the solution process.

Keywords: Cognitive Load Theory; Error Diagnosis; Integral Technique; Newman's Error Analysis.

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Introduction

Integrals are mathematical objects with high epistemic complexity that have many interrelated conceptual dimensions, but are often taught separately in class (Mateus-Nieves & Moll, 2021; Özgeldi & Aydın, 2021). In practical learning, students often experience difficulty in distinguishing integral solution techniques, for example, between the substitution method, partial methods, and other techniques. Difficulty in selecting a technique often leads to various types of errors, both conceptual and procedural (Özgeldi & Aydın, 2021; Sumargiyani et al., 2021). Recent studies also show that students struggle to connect integral concepts with underlying mathematical representations and prerequisite knowledge, causing fragmented understanding during problem solving (Oktaviyanthi & Agus, 2025; Parveen, 2025; A. D. Rahmawati et al., 2024). Integrals are rooted in the need to calculate quantities such as area, volume, and length of curves through the process of summation limits, which are seen as a generalization of the idea of infinite summation, which was later formalized in the Riemann definition and further developed with various other approaches (Thomson, 2013). Despite this, students often face confusion in understanding the concept of integrals (Kidron, 2020).

The results of the study indicate that students' learning difficulties in integrals are caused by weak mastery of prerequisite material (precalculus), lack of understanding of integration concepts, low self-confidence, and the need for more study time and practice, where integration techniques such as substitution, partial (integration by parts), and partial fractions are the most difficult parts for students (Angco, 2021). According to Meika et al. (2023), the biggest types of errors made by students in integral calculus are conceptual errors (56.25%), followed by procedural errors (54.17%) and computational errors (45.83%), which are generally caused by limited understanding of concepts, difficulty translating mathematical symbols, a tendency to memorize formulas, and inaccuracy in work. Nurhasanah & Dollo (2021) concluded that student errors in solving areas with integration include conceptual errors (e.g., inverse integration limits or incorrect graphs), procedural errors (incorrect calculations), and errors in the final solution (answers that do not match the question). Recent studies further emphasize that students tend to memorize formulas mechanically without understanding when and why a particular technique should be applied (Parveen, 2025; A. D. Rahmawati et al., 2024). These findings indicate that students' difficulties in integral calculus lie in the application of solution techniques, which are rooted in weak conceptual understanding and prerequisite skills, thus having implications for the emergence of various types of errors in the integral solution process.

In high school, integrals are introduced as antiderivatives as per the historical tradition in the 17th-18th centuries, while in college, the Riemann integral is introduced as the limit of summation, where modern teaching is often reversed from the historical chronology, which complicates students' conceptual understanding (Bressoud, 2011). Consequently, many students experience difficulties when dealing with problems involving area, volume, improper integrals, or accumulation contexts that require deeper conceptual interpretation (Khounphia et al., 2020; Nurhayati, Priatna, et al., 2023; R. D. Rahmawati et al., 2019). This means that many students have difficulty understanding integrals because they are immediately introduced to the symbols and procedures of calculus, without truly understanding the conceptual meaning of integrals as an accumulation concept (Kouropatov & Dreyfus, 2014). The limitations of conceptual understanding make them tend to rely solely on algorithmic procedures, making them prone to errors when facing variations in integral problem forms.

The conceptual gap in integral learning, where students are only introduced to integrals as the "inverse of the derivative" since high school, narrows their mental schema and limits the development of richer mental models of integrals that connect geometric, symbolic, and accumulation meanings (Greefrath et al., 2021). This aligns with the findings of Yohanes & Yusuf (2021), who showed that mathematical problem-solving is strongly influenced by students' cognitive maps, namely how they organize knowledge, procedures, concepts, and problems. Students' processes of absorbing information to form knowledge can create cognitive load during learning or when solving mathematical problems (Richardo & Cahdriyana, 2021). The cognitive load that arises from each interconnection can explain why students often have difficulty choosing the right integral technique and are prone to making conceptual and procedural errors.

Cognitive load theory (CLT) was introduced in the 1980s based on human cognitive architecture. According to Sweller et al. (2019), CLT is based on human cognitive architecture, emphasizing the cognitive processing demands during problem solving in processing new information and the crucial role of long-term memory in learning. The development of this theory emphasizes the importance of reducing extraneous cognitive load (de Jong, 2010) and optimizing the allocation of cognitive resources to build knowledge in long-term memory. Initially, CLT divided cognitive load into three types: intrinsic, extraneous, and germane load. First, intrinsic load, which is the load that comes from the complexity of the subject matter and the interactivity of the elements that must be understood. Second, extraneous load, which is the additional load caused by the presentation of information or less efficient learning design (Sweller, 2010). Meanwhile, the concept of German load, previously viewed as a separate type of load, is now better understood as part of cognitive resource management aimed at effectively processing intrinsic load and supporting the formation of schemas (Kalyuga, 2011). Therefore, the core of CLT lies in efforts to minimize extraneous load and balance intrinsic load according to students' working memory capacity so that learning becomes more optimal.

As a step to understand students' learning difficulties more deeply, error diagnosis is an important approach. One theory regarding error analysis or diagnosis is Newman's Error Analysis (NEA). Newman argues that to successfully solve a written math problem, a child must be able to read the words, understand their meaning, transform verbal information into appropriate mathematical operations, apply correct process skills, and finally present the results in an acceptable written form (Newman, 1977). The stages in solving math problems based on NEA consist of: (1) reading error; (2) comprehension error; (3) transformation error; (4) process skill error; and (5) encoding error, where Newman's framework provides the highest overall performance compared to other models such as Kastolan, Watson, Hadar, and Polya, mainly due to its structured approach to error analysis and its application in the context of formative assessment (Garcia Tobar et al., 2025). Errors can occur at any of these stages, and each type of error provides important diagnostic information for educators. Similar research that conducted error diagnosis using NEA on integral calculus material is research by Nurrasa & Nurrohman (2024) regarding difficulties in using substitution and integration by parts techniques, and errors that were frequently found included the Reading stage, Transformation stage, Process Skill stage, and Encoding stage.

Previous studies have examined students' errors in integral calculus using NEA and separately investigated students' cognitive difficulties through CLT. However, studies integrating NEA and CLT to explain how specific error types reflect different cognitive load components remain limited, particularly in the context of distinguishing integral solution techniques. Most prior studies focus primarily on identifying procedural or conceptual errors without explaining the cognitive mechanisms underlying the emergence of those errors. In particular, transformation errors related to selecting appropriate integration techniques have rarely been examined from a cognitive load perspective. From the perspective of CLT, difficulties in selecting integral techniques and the tendency to make conceptual and procedural errors may arise from high intrinsic cognitive load caused by the complexity and interconnectedness of integral concepts, as well as extraneous cognitive load resulting from procedural-oriented instruction and inefficient presentation of information. These conditions can overload students' cognitive resources during problem solving, thereby increasing the likelihood of errors in determining and applying integral techniques. Therefore, this study offers a novel contribution by integrating NEA and CLT to provide a more comprehensive explanation of students' errors in solving integral calculus problems, particularly in relation to intrinsic, extraneous, and germane cognitive load. Based on these considerations, this study aims to analyze students' errors in solving integral calculus problems using NEA and interpret the findings through the perspective of Cognitive Load Theory.

Research Methods

This study aims to determine the errors made by students in answering questions on the Integral Calculus material based on Newman's Error Analysis (NEA) and to view it from the perspective of cognitive load theory (CLT). This type of research is descriptive research with a quantitative approach. The population consisted of 74 students from three classes. The sampling technique used was purposive sampling, with the consideration that the research subjects must be students who have just received integral material in the calculus course. Therefore, students of the class of 2024 were selected as the research sample, because they have met the appropriate criteria for identifying the types of errors that occur in differentiating integral solution techniques, ensuring similar lecturers, class conditions, as well as the similarity of learning time and schedule for changing class hours.

The data obtained were quantitative, in the form of an integral calculus test, which consisted of five open-ended integral calculus problems representing different integral solution techniques. The instrument was validated through expert judgment by two mathematics education lecturers specializing in calculus and mathematics evaluation. In addition, empirical validity testing showed that all item correlation coefficients exceeded the minimum validity threshold ($r > r_{table} = 0.229$) with significance values below 0.05, indicating that all test items were valid. Reliability testing using Cronbach's Alpha produced a coefficient of 0.786, which exceeded the acceptable reliability standard ($\alpha > 0.70$). Therefore, the instrument was considered reliable and consistent in identifying students' errors in solving integral calculus problems. The test results were analyzed using descriptive statistical analysis, including calculating student answer scores, grouping errors based on the NEA framework, tabulating the frequency and percentage of each type of error, and calculating the proportion and standard deviation of the proportion. The results of the analysis were presented in a table to identify the types of student errors in solving integral problems.

The proportion was calculated using the following formula.

$$p = \frac{x}{n}$$

Where x is the number of students who made errors in an integral solution technique, and n is the total number of students analyzed.

The standard deviation of the proportion is calculated using the formula

$$\sigma = \sqrt{p(1-p)}$$

both of which are based on the binomial distribution theory (Walpole et al., 2017). The proportion is used to show the ratio of students who make errors in each integral technique, while the standard deviation value is used to see the diversity of error proportions between techniques.

The interpretation of proportion values is carried out based on categories adapted from the descriptive analysis guidelines by Riduwan (2009) and Sugiyono (2014) as shown in Table 1 below.

Table 1. Interpretation of Proportional Values

Proportional Interval (p)	**Interpretation**
$p \geq 0.30$	High
$0.10 \leq p < 0.30$	Medium
$p < 0.10$	Low

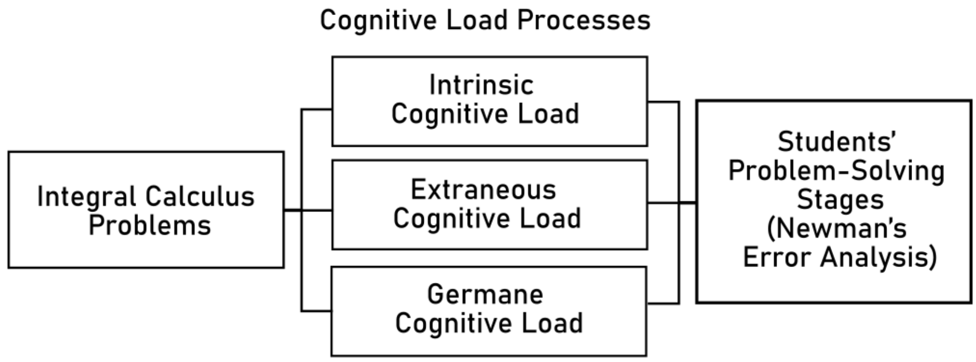

Figure 1. Conceptual Framework Integrating NEA and CLT in Integral Calculus Problem Solving

Figure 1 presents the conceptual framework of this study, integrating Newman's Error Analysis (NEA) and Cognitive Load Theory (CLT). The framework assumes that students' errors in solving integral calculus problems emerge from the interaction between the cognitive demands of integral concepts and the cognitive processing demands during problem solving. Intrinsic cognitive load originates from the complexity of coordinating multiple representations and selecting appropriate integration techniques, whereas extraneous cognitive load arises from procedural-oriented instruction and inefficient information processing. Meanwhile, germane cognitive load supports schema construction and conceptual integration. These cognitive processes are reflected in different stages of NEA, particularly transformation and process skill errors, which become dominant in integral calculus learning.

This diagnosis is then used to examine the potential cognitive load that arises from a CLT perspective, thus providing a more comprehensive picture of students' difficulties in distinguishing integral solution techniques. Table 2 shows the error types and error indicators based on the NEA (Garcia Tobar et al., 2025).

Table 2. Error Indicators Based on Newman's Error Analysis (NEA)

Error Type	Error Indicator
Reading Error	Failed to identify important information in the question. Mistakes in determining known data. Using homemade symbols without explaining their meaning.
Comprehension Error	Answering incorrectly due to a lack of understanding or incomplete identification of the question elements. Writing short but unclear answers, and lacking adequate argumentation in conveying what needs to be resolved.
Transformation Error	Mistakes in selecting and using the right technique. Inaccuracies in converting information into mathematical formulas
Process Skill Error	Errors in using arithmetic operations. Procedures/settlement steps are incomplete.
Encoding Error	Incorrect answer writing. The answer does not fit the context. The conclusion is inaccurate or inconsistent.

Results and Discussions

Students' difficulties in solving integral calculus problems can be seen in the types of errors they make. This section contains the results of research on students' errors in distinguishing integral solving techniques from the perspective of Cognitive Load Theory (CLT). Data were obtained from the analysis of student answer sheets using error classification according to Newman's Error Analysis (NEA), which includes reading errors, comprehension errors, transformation errors, process skill errors, and encoding errors. The following is a description of the research results based on the type of NEA error.

Reading Error

Based on the analysis, no students made any errors in the reading error type. This indicates that all students were able to correctly read and copy the information provided in the problem. Therefore, the errors that appeared in solving integrals did not originate from the initial reading stage of the problem, but rather occurred more predominantly at other stages of the NEA.

Comprehension Error

Based on the analysis, comprehension errors occurred at a relatively low level overall, indicating that most students were able to identify the information and objectives presented in the problems. This finding suggests that students generally did not experience major obstacles at the initial stage of understanding the problem context.

However, the distribution of errors across problem types reveals that comprehension difficulties were more likely to emerge in problems requiring conceptual interpretation rather than procedural recall. A detailed distribution of comprehension errors is presented in Table 3.

Table 3. Descriptive Statistics of Comprehension Error Proportion

Integral Solving Techniques	Frequency	Percentage	p	σ	Interpretation
Integration by parts	1	1.35%	0.0135	0.116	Low
Integration using partial fractions	2	2.7%	0.0270	0.163	Low
Improper integrals	7	9.46%	0.0946	0.291	Low
Integral of the area of a region	7	9.46%	0.0946	0.291	Low
Volume using the disk method	24	32.43%	0.3243	0.468	High

The highest comprehension error occurred in volume integral problems using the disc method (32.43%). This pattern indicates that students struggled not merely with interpreting the wording of the problem, but with understanding the conceptual relationship between geometric representation, axis rotation, and the resulting integral model. Students tended to recognize the formula mechanically without fully understanding why a particular radius or boundary should be used. This finding suggests that comprehension errors in integral calculus are closely related to weak conceptual visualization and insufficient integration between graphical and symbolic representations.

In contrast, comprehension errors in integration by parts problems were very low. This may indicate that students were more familiar with routine procedural tasks involving algebraic manipulation than with application-based integral problems requiring conceptual reasoning. Similarly, improper integrals and area integrals produced lower comprehension errors because students were generally able to recognize the problem structure, even though difficulties emerged in later stages of the solution process. Therefore, comprehension errors in this study were not primarily caused by failure to read or identify information, but rather by limited conceptual understanding in non-routine and application-oriented integral problems.

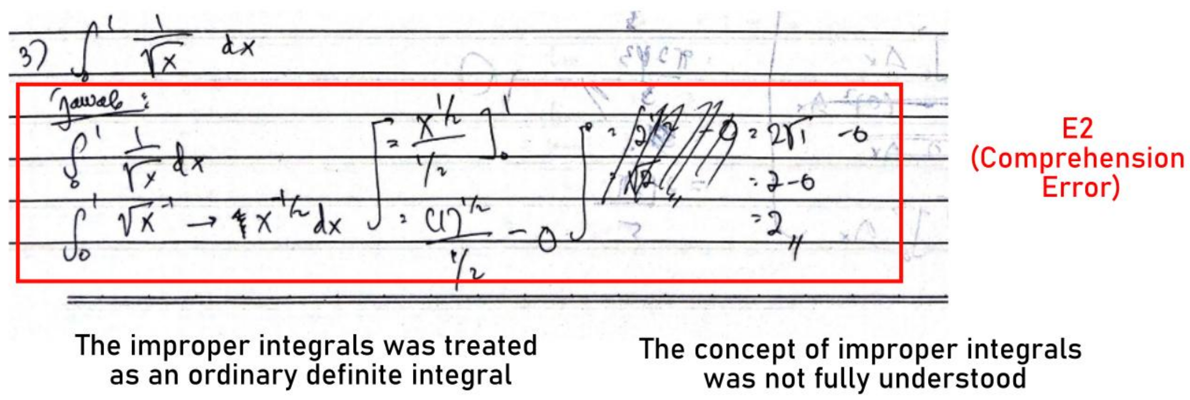

Figure 2. Representative student response showing a comprehension error

Figure 2 presents a representative student response categorized as a comprehension error. The student directly evaluated the improper integral as an ordinary definite integral without rewriting it into limit form. This response indicates that the student did not fully understand the conceptual nature of improper integrals, despite obtaining the correct numerical result.

Transformation Error

Transformation error was the most dominant type of error found in this study. This indicates that although students generally understood the problem statement, many failed to transform that understanding into an appropriate mathematical strategy or integral technique. Thus, the main difficulty was not recognizing what was being asked, but determining how the problem should be solved mathematically. A more complete breakdown is presented in Table 4.

Table 4. Descriptive Statistics of Transformation Error Proportion

Integral Solving Techniques	Frequency	Percentage	p	σ	Interpretation
Integration by parts	50	67.57%	0.6757	0.468	High
Integration using partial fractions	40	54.05%	0.5405	0.499	High
Improper integrals	50	67.57%	0.6757	0.468	High
Area of a plane region	49	66.22%	0.6622	0.473	High
Volume using the disk method	19	25.68%	0.2568	0.437	Medium

Transformation errors were highest in the integration by parts problem and improper integrals (67.57%). This finding indicates that students experienced substantial difficulty in identifying the structural characteristics of a problem and associating them with an appropriate integration technique. Many students directly applied familiar procedures without analyzing whether the selected method satisfied the mathematical conditions of the problem. For example, students frequently used substitution techniques in situations requiring integration by parts or failed to recognize the role of limits in improper integrals.

A similarly high proportion was also found in the area of a plane region (66.22%), suggesting that students struggled to translate geometric information into mathematical models. In many cases, students could identify the boundary curves but failed to determine which function should be subtracted or integrated with respect to the appropriate variable. This indicates that transformation errors were strongly influenced by weak connections between conceptual understanding and procedural decision-making.

Interestingly, transformation errors in volume integrals were lower than in other techniques. This may indicate that students relied more heavily on memorized formulas in disc-method problems, allowing them to proceed procedurally even without deep conceptual understanding. Overall, the findings suggest that transformation errors emerge when students are required to select among multiple competing techniques and construct solution pathways independently. Therefore, the issue lies not only in procedural knowledge, but also in students' inability to organize conceptual relationships among integral representations, techniques, and problem structures.

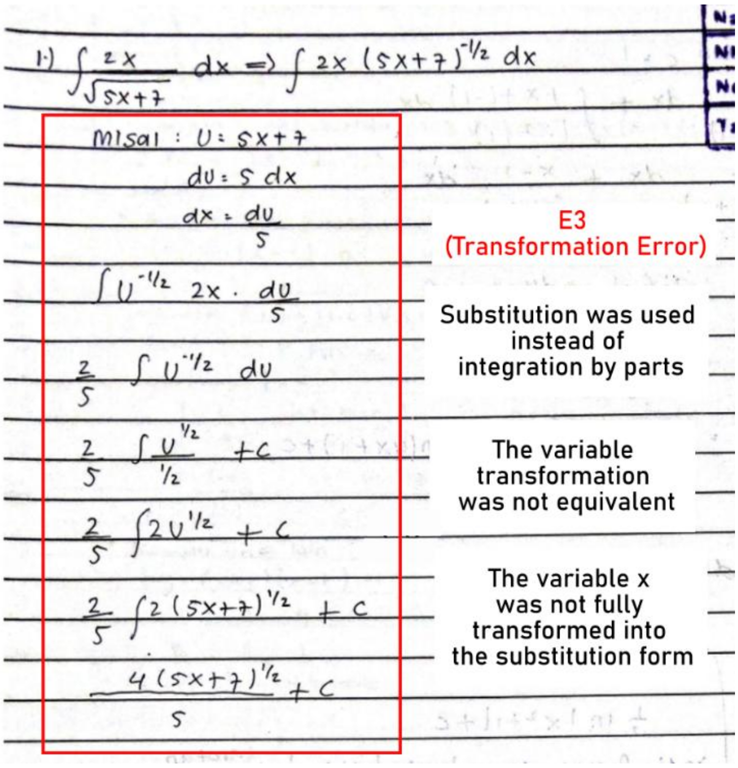

Figure 3. Representative student response showing a transformation error

Figure 3 illustrates a transformation error in solving an integration by parts problem. Although the problem explicitly instructed the use of integration by parts, the student applied the substitution method instead and performed an incomplete variable transformation. This response demonstrates difficulty in selecting and constructing an appropriate solution strategy.

Process Skill Error

Process skill errors occurred when students failed to correctly execute algebraic manipulations, arithmetic operations, or procedural steps after selecting a solution method. Unlike transformation errors, which are related to strategy selection, process skill errors reflect weaknesses in procedural fluency and accuracy during the execution stage. A detailed distribution is shown in Table 5.

Table 5. Descriptive Statistics of the Proportion of *Process Skill Errors*

Integral Solving Techniques	Frequency	Percentage	p	σ	Interpretation
Integration by parts	21	28.38%	0.2838	0.451	Medium
Integration using partial fractions	28	37.84%	0.3784	0.485	High
Improper integrals	10	13.51%	0.1351	0.342	Medium
Area of a plane region	17	22.97%	0.2297	0.420	Medium
Volume using the disk method	9	12.16%	0.1216	0.329	Medium

The highest process skill error occurred in rational function integrals (37.84%). This finding indicates that students had difficulty carrying out multi-step algebraic manipulations, particularly in decomposing rational expressions into partial fractions before integration. Errors frequently involved incorrect factorization, inaccurate coefficient determination, or incomplete decomposition procedures. These mistakes suggest that students' prerequisite algebraic knowledge was insufficiently automated, causing working memory resources to be overloaded during problem solving. Process skill errors also appeared in integration by parts (28.38%). In these problems, students often selected the correct technique but made procedural mistakes when applying differentiation and integration repeatedly. This pattern indicates that procedural execution became unstable when students were required to coordinate several sequential operations simultaneously.

Although process skill errors in improper and volume integrals were relatively lower, they still demonstrate difficulties in handling advanced procedural operations, such as evaluating infinite limits or manipulating quadratic expressions in geometric applications. These findings imply that procedural errors are not solely caused by carelessness, but also by cognitive overload during the execution stage, especially when students must simultaneously manage symbolic manipulation, intermediate calculations, and procedural consistency.

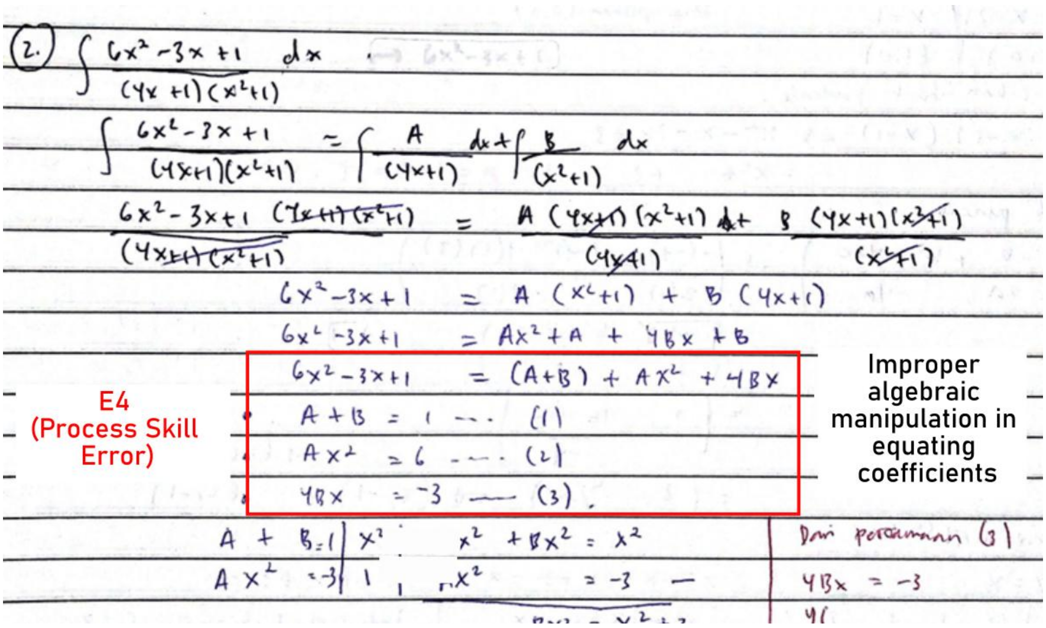

Figure 4. Representative student response showing a process skill error

Figure 4 shows a process skill error in solving a rational function integral using partial fraction decomposition. The student selected the correct general method but made errors in equating coefficients and algebraic manipulation, leading to an incorrect decomposition process.

Encoding Error

Based on the analysis, no students made any errors in the encoding error type. This indicates that all students were able to write conclusions that aligned with their results. In other words, students did not experience any difficulties in communicating solutions in the form of correct mathematical conclusions.

Table 6. Average Percentage of Errors

Error Type	Average Percentage	p	σ	Interpretation
Reading Error	0%	0.0000	0,000	Low
Comprehension Error	11.08%	0.1108	0.314	Medium
Transformation Error	56.22%	0.5622	0.496	High
Process Skill Error	22.97%	0.2297	0.420	Medium
Encoding Error	0%	0.0000	0,000	Low

Based on Table 6, it can be seen that the dominance of transformation errors, combined with relatively low comprehension errors, suggests that students' primary difficulty does not lie in understanding problem statements, but in coordinating conceptual and procedural knowledge during strategy selection. The coexistence of high transformation errors and moderate process skill errors indicates that students' failures in integral calculus are not purely procedural, but emerge from overloaded working memory during conceptual decision-making.

Student Errors in Solving Integral Calculus Problems

The analysis results show that the dominant student errors are transformation errors, namely errors in converting problem understanding into a mathematical model that is appropriate for the solution. These errors are primarily related to selecting the appropriate integral technique. Although most students have understood the purpose of the problem, many of them experience confusion in determining whether the problem is more appropriately solved by substitution, partial fractions, or even geometric techniques (Nurrasa & Nurrohman, 2024; Susilo et al., 2021). Students have difficulty with aspects of the concept, procedure, and problem solving of integrals, including various techniques and methods (Nursyahidah & Albab, 2017). This shows that distinguishing integral techniques is still often confusing for students.

Transformation errors occur when students fail to select appropriate formulas or plan the correct solution steps, and if the transformation steps are wrong, the error will "follow" to the next stage, namely the skill process (Angco, 2021). Several similar studies have found that students often make mistakes in determining the correct technique, with a total error of 42.8% according to Li et al. (2017). Errors in definite integrals are caused by choosing the wrong technique when transforming the problem into a workable integral form (Darvishzadeh et al., 2019). In addition, the tendency to use the usual procedure without in-depth analysis of the problem requirements is the cause of transformation errors (Machromah & Purnomo, 2017). This confirms that the main difficulty for students is using the correct integral method.

In addition to transformation errors, procedural errors are often made by students when solving integrals. Process skill errors occur when difficulties with algebraic manipulations occur, including errors in basic arithmetic operations. This type of error is also frequently made by students when solving integral calculus problems (Machromah & Purnomo, 2017). Furthermore, a lack of understanding of basic precalculus concepts leads many students to make procedural errors when solving integral calculus (Moradi et al., 2023). In line with research by Mahadewsing et al. (2024), weak mastery of prerequisite knowledge (algebra and trigonometry) is a dominant factor triggering procedural errors in integral calculus. Inaccuracy and an inadequate understanding of basic mathematical concepts are common causes of these errors.

Misunderstanding problems or comprehension errors occur when students change the form of mathematical sentences in problems to become simpler and can be easily integrated (Machromah & Purnomo, 2017). In this study, comprehension errors were also quite high, where students did not know the steps to take when they had understood the information in the problem, such as in the problem of determining volume using the disc method to find the area of a circle, which was caused by understanding the concepts related to integral applications that needed to be further honed. This is in line with research by Kusumaningrum et al. (2020), which found that in problems determining volume, students experienced errors in determining integral functions, integral limits, and volumetric solution techniques. This technique requires a combination of geometric and algebraic concepts, so it is quite complex.

When it comes to reading questions, students generally tend to have low error rates, as found in this study. The same finding was found by Valdez & Taganap (2024), where reading errors were in the low category. Similarly, encoding errors were not found in this study, as was found by Machromah and Purnomo (2017), as students were generally able to write conclusions based on their results.

Based on previous findings and studies, students' errors in solving integral problems can be grouped into three main factors: (1) weak mastery of prerequisite concepts such as algebra and trigonometry; (2) learning that emphasizes procedures without conceptual meaning; and (3) limited metacognitive strategies in reviewing solution steps. These three factors are interrelated and have an impact on transformation and procedural errors. The error patterns that emerge indicate a high cognitive load in the process of solving integral problems. Therefore, further analysis was conducted using the Cognitive Load Theory (CLT) framework to further examine the sources and types of cognitive load experienced by students.

Student Errors in Solving Integral Calculus Problems: A Cognitive Load Theory (CLT) Perspective

The results of the analysis of students' errors in solving integral calculus problems cannot be separated from the cognitive load they experience. When viewed from the distribution of errors, students most often made transformation errors (56.22%), followed by process skill errors (22.97%), and a small portion in the form of comprehension errors (11.08%), while reading errors and encoding errors were not found. This error pattern can be explained through the Cognitive Load Theory (CLT) framework, which distinguishes cognitive load into intrinsic cognitive load, extraneous cognitive load, and germane cognitive load. (Sweller, 2010; Sweller et al., 2019). The following is a description of student errors and their relationship to CLT in Table 7 below.

Table 7. Description of Student Errors and Their Relation to Cognitive Load Theory (CLT)

Error Type	Primary Cognitive Load	Description
Reading Error	Low	Students are able to read questions well, with minimal cognitive load.
Comprehension Error	Germane cognitive load (medium)	Students are able to understand the problem and construct a solution scheme

Error Type	Primary Cognitive Load	Description
		conceptually, but have difficulty understanding the meaning of the problem, which is not directly related to antiderivatives.
Transformation Error	Intrinsic cognitive load (high) & extraneous cognitive load (high)	Students have difficulty choosing the right technique due to the complexity of the integral material and learning approaches that emphasize mechanical procedures without conceptual context.
Process Skill Error	Intrinsic cognitive load (high) & extraneous cognitive load (high)	Students have difficulty in algebraic manipulation and completing the steps of the integral procedure they have chosen, and their working memory is burdened by non-essential information.
Encoding Error	Low	Students are able to write down the results of their calculations, with minimal cognitive load.

First, the high intrinsic cognitive load in integral material triggers transformation errors and process skill errors in solving integral calculus problems. Integral material, particularly in distinguishing solution techniques (e.g., integration by parts, substitutions, improper integrals, and volume methods), is naturally highly complex. Students must recognize problem types, relate them to relevant solution strategies, and memorize formal integral rules. Intrinsic cognitive load, it is the cognitive load that arises from the inherent and unchangeable complexity of content or learning materials (Gupta & Zheng, 2020). The high complexity of integral material is a major factor in the emergence of transformation errors. The high cognitive load at this stage causes many students to choose the wrong method, even though they actually understand the intent of the problem.

Process skill errors also illustrate that students fail to manage cognitive load when processing information. Although students are able to determine the method, errors often occur in algebraic procedures, trigonometric calculations, and symbol manipulation. This difficulty may arise because students struggle to efficiently manage procedural steps, resulting in increased memory workload due to the presence of non-essential information. This is related to poorly managed extraneous cognitive load (Sweller et al., 2019). In any teaching situation, intrinsic cognitive load and extraneous cognitive load cannot both be high at the same time.

The relatively small comprehension error indicates that the majority of students were able to understand the problem text. This low percentage can be interpreted as meaning that the German cognitive load, namely the allocation of cognitive resources to construct a solution scheme, was working quite well at the comprehension stage. However, when they had to proceed to the strategy transformation stage, students' cognitive capacity was again burdened, resulting in persistent errors. The absence of reading and encoding errors indicates that students did not experience difficulties in the initial reading stage or in the final stage of writing down calculation results. This means that the main errors did not lie in input or output, but rather in information processing during the solution process.

The high complexity of integral calculus material is inseparable from how integrals are taught since high school. According to (Bressoud, 2011), the majority of college calculus students are already familiar with the definition of integrals, which is firmly embedded as anti-differentiation since high school. The historical, symbolic, and conceptual meaning of integrals is not fully understood by students (Nilsen & Knutsen, 2023). This condition creates a high intrinsic cognitive load because students must reconstruct the meaning of integrals when they are faced with integral problems that are not directly related to anti-derivatives, such as improper integrals, area integrals, or volume integrals (Khounphia et al., 2020; Nurhayati, Suryadi, et al., 2023; R. D. Rahmawati et al., 2019). Therefore, learning integral calculus in schools requires an understanding beyond simply "technical formulas" that are considered mechanical without recognizing the conditions under which certain techniques are valid.

Teaching approaches that focus on mechanical formulas without conceptual context also increase extraneous cognitive load (Leung et al., 1997), because students must memorize technical steps without understanding the conceptual reasoning behind them. Students are rarely allowed to build an understanding that an integral is an accumulation function that adds small changes to a total (Palha & Spandaw, 2019). Furthermore, teaching often overemphasizes procedural and symbolic aspects, with minimal assignments that require relationships between concepts (García-García et al., 2025). Students accustomed to a single representation struggle with problems that require multiple representations (Dinçer, 2022). However, integrals as mathematical objects have undergone several evolutionary periods, from geometric to analytical definitions, and connecting the geometric, symbolic, and procedural meanings of integrals can improve students' understanding (Mateus-Nieves & Moll, 2021). In other words, intrinsic cognitive load arises from the complexity of the integral material itself, while extraneous cognitive load arises from teaching approaches that overemphasize mechanical procedures without conceptual context.

Providing opportunities for learners and students to connect various integral representations systematically helps them optimize the germane cognitive load, namely the ability to build and strengthen integral conceptual schemes, so that understanding of concepts becomes deeper and reduces confusion due to wrong interpretations from the early stages of learning. As an effort to minimize extraneous load, namely the additional burden caused by the presentation of information or learning design that is less efficient, educators need to present integral calculus material according to its essence and history of development, so that students can understand integral concepts in depth and reduce confusion due to wrong interpretations from the early stages of learning. Overall, the findings of this study confirm that students' errors in solving integral calculus problems are influenced not only by limited conceptual and procedural mastery but also by cognitive load management during the learning process. The dominance of transformation and process skill errors indicates that high intrinsic cognitive load and inefficiently managed extraneous cognitive load substantially affect students' ability to solve integral problems, while limited connections between geometric, symbolic, and procedural meanings restrict the optimization of germane cognitive load (Greefrath et al., 2021).

This study contributes theoretically by proposing an interpretive relationship between NEA error stages and cognitive load components in integral calculus learning.

The findings suggest that transformation errors predominantly reflect high intrinsic cognitive load related to selecting appropriate integration techniques, whereas process skill errors are associated with inefficient management of extraneous cognitive load during procedural execution. Furthermore, this study extends CLT by showing that transformation errors emerge when students must coordinate multiple mathematical representations and determine solution strategies simultaneously. It also refines the application of NEA in higher mathematics by linking each error stage with different sources of cognitive load. Despite these contributions, several limitations should be acknowledged. This study was limited to students from one institution and relied solely on written responses without interviews or think-aloud protocols, so cognitive processes were inferred from students' answer patterns rather than directly observed. In addition, the study focused only on several integral techniques and may not fully represent all dimensions of integral calculus learning.

Conclusions and Suggestions

Based on the findings, the most dominant errors in solving integral calculus problems were transformation errors, followed by process skill and comprehension errors, while reading and encoding errors were not found. Transformation errors indicate that students experienced difficulties in selecting appropriate integration techniques, whereas process skill errors reflected weaknesses in algebraic manipulation and procedural execution. From the perspective of Cognitive Load Theory (CLT), these errors were influenced by high intrinsic cognitive load arising from the complexity of integral concepts and by extraneous cognitive load caused by procedural-oriented instruction. Meanwhile, the relatively low comprehension errors suggest that students were generally able to understand problem statements, although they still struggled to coordinate conceptual understanding and procedural decision-making.

The pedagogical implications of these findings highlight the importance of instructional strategies that manage intrinsic cognitive load, minimize extraneous cognitive load, and support the development of conceptual schemas through germane cognitive load. These strategies may include scaffolding, structured prerequisite exercises, and presenting integral calculus as an accumulation function rather than merely a collection of procedural formulas. Before introducing integration techniques, lecturers should explicitly compare the structural characteristics of functions that require substitution, integration by parts, or partial fractions. Such comparisons can help students recognize conceptual differences between techniques and reduce unnecessary cognitive load during strategy selection. In addition, for volume-integral problems, students should first construct graphical representations before forming symbolic integral expressions, allowing conceptual connections between geometric and algebraic representations to develop gradually. Through these approaches, students can allocate cognitive resources more effectively, deepen conceptual understanding, and improve their ability to select appropriate solution techniques.

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