Alifmatika: Jurnal Pendidikan dan Pembelajaran Matematika
Volume 8, Issue 1, 135-162, June 2026
e-ISSN: 2715-6109 | p-ISSN: 2715-6095
https://journal.ibrahimy.ac.id/index.php/Alifmatika

Mapping students' mathematical representation trajectories across five habits of mind performance levels in similarity and congruence problems

Muhammad Razali[1*], Cut Latifah Zahari[2], Siti Shara Siagian[3]

[1*]Faculty of Economy, Universitas Pembinaan Masyarakat Indonesia, Sumatera Utara 20214, Indonesia
[2,3]Mathematics Education, Universitas Muslim Nusantara Al-Washliyah, Sumatera Utara 20147, Indonesia
[1*]razalialy@gmail.com, [2]cutlatifah@umnaw.ac.id, [3]sharasiagian123@gmail.com

Received: September 30, 2025 | Revised: June 14, 2026 | Accepted: June 19, 2026 | Published: June 25, 2026
*Corresponding author

Abstract:
This study aims to map students' trajectories of mathematical representation across five Habits of Mind performance levels in solving similarity and congruence problems. Unlike previous descriptive studies, which generally examine the relationship between mathematical representation ability and Habits of Mind, this study emphasizes how students' pictorial, symbolic, and verbal representations develop, intersect, and differ across performance levels. A descriptive qualitative approach was employed involving 30 ninth-grade junior high school students selected through purposive sampling. The data were collected using a mathematical representation ability test, a Habits of Mind questionnaire, and semi-structured interviews. The findings reveal five representation trajectories. At the beginner level, students' representations were fragmented and lacked meaningful connections to the problem. At the limited level, representations began to appear but remained partial, inconsistent, and procedurally weak. At the developing level, students showed emerging ability to connect pictorial, symbolic, and verbal forms, although conceptual errors still occurred. At the proficient level, students demonstrated more structured symbolic and pictorial representations, but their written verbal explanations still required refinement. At the superior level, students were able to coordinate representations flexibly, systematically, and meaningfully. A notable finding is that representation development does not always progress linearly; some students at the developing level showed representational characteristics close to the proficient level. Therefore, this study contributes a five-level trajectory map that explains the dynamics of students' mathematical representation ability through the lens of Habits of Mind performance.

Keywords: Habits of Mind Performance; Mathematical Representation Trajectory; Pictorial Representation; Similarity and Congruence Problems; Symbolic Representation; Verbal Representation.

How to Cite: Razali, M., Zahari, C. L., & Siagian, S. S. (2026). Mapping students' mathematical representation trajectories across five habits of mind performance levels in similarity and congruence problems. *Alifmatika: Jurnal Pendidikan dan Pembelajaran Matematika*, 8(1), 135-162. https://doi.org/10.35316/alifmatika.2026.v8i1.135-162

Introduction

Mathematical representation is one of the essential abilities that needs to be developed in mathematics learning. In line with the basic framework of the independent

Content from this work may be used under the terms of the Creative Commons Attribution-ShareAlike 4.0 International License that allows others to share the work with an acknowledgment of the work's authorship and initial publication in this journal.
Copyright (c) 2026 Alifmatika: Jurnal Pendidikan dan Pembelajaran Matematika

curriculum and the National Council of Teachers of Mathematics (NCTM), the basic abilities that need to be developed in mathematics learning include mathematics as problem solving, communication, reasoning, connections, and representation (Haryadi et al., 2024; Siagian et al., 2022; Wahyuni et al., 2022). Among these abilities, representation plays an important role because it enables students to express mathematical ideas, translate problem situations into mathematical forms, and communicate their thinking in various ways. When students have strong mathematical representation ability, they can solve mathematical problems more flexibly and do not depend on a single solution strategy (Mujib, A., Firmansyah, Siagian, S. S., 2025; Mujib, 2022; Siagian et al., 2022).

The urgency of developing mathematical representation ability is further underscored by the results of the 2022 Program for International Student Assessment (PISA). Indonesia recorded a mathematics score of 366 in PISA 2022, a decline from 379 in 2018, placing the country 70th out of 81 participating nations (OECD, 2022). More critically, more than 60% of Indonesian students scored at Level 1 or below in mathematics, a level characterized by very limited basic skills, in which students can only understand simple concepts without being able to apply them or interpret them in broader, more complex contexts (OECD, 2022). Moreover, PISA 2022 data reveal that approximately 81.7% of Indonesian students perform below the minimum proficiency level (Level 2) in mathematics, one of the highest proportions among all participating countries (OECD, 2022). A particularly relevant finding for the present study is that Indonesia also registered one of the lowest shares of students who feel confident in extracting mathematical information from diagrams, graphs, or simulations, at only 47% (OECD, 2022). These figures collectively indicate that Indonesian students face fundamental challenges not only in procedural computation but also in mathematical representation, particularly in interpreting, constructing, and using pictorial and symbolic forms to communicate mathematical ideas.

Mathematical representation ability refers to the ability to present or rewrite mathematical ideas in the form of symbols, diagrams, graphs, tables, equations, or other mathematical expressions (Choiriyaza & Fatma, 2021; Haryadi et al., 2024; Mustangin, 2015). Mathematical representation generally includes visual, verbal, and symbolic representations (Ikhsan et al., 2024; Lubis et al., 2024; Zahari & Razali, 2022). Visual representation involves the ability to create images, diagrams, or graphs to clarify problems and support problem-solving. Verbal representation refers to the ability to state mathematical ideas, write problem-solving steps, and explain the interpretation of a representation. Symbolic representation refers to the ability to create mathematical models and solve problems involving mathematical expressions (Choiriyaza & Fatma, 2021; Ikhsan, Dewi, & Waluya, 2024; Mujib, 2022; Siregar, 2021). Thus, representation is not only a final product of students' answers but also a thinking tool that helps students understand, organize, and solve mathematical problems.

In mathematics learning, representation is closely related to students' ability to understand contextual problems. A problem can be represented by objects, pictures, words, or mathematical symbols (Hasnarika, 2022). Therefore, representation becomes an important communication and thinking tool in solving mathematical problems. Mathematical representation is also needed in communicating mathematical ideas in certain ways (Malasari, Herman, & Jupri, 2019). In addition, mathematical representation can be understood as a way of expressing mathematical ideas that shows students' thinking as a result of their interpretation of a problem situation (Hilmi et al., 2023). It indicates that representation is important and necessary for students to understand mathematical material and solve problems (Hasnarika, 2022; Hilmi et al., 2023).

One topic that requires strong mathematical representation ability is similarity and congruence. In solving similarity and congruence problems, students need to identify information from the problem, create appropriate images, determine corresponding sides, use proportional relationships, and explain their reasoning. However, students often experience difficulties when they have to transform verbal information into pictorial and symbolic forms. Some students can draw a figure, but the figure lacks meaningful information; some can write mathematical symbols, but the symbols are not connected to the given problem; and some can give verbal answers, but the explanations are not conceptually accurate. These difficulties show that mathematical representation ability is not only related to students' mathematical knowledge but also to how students think, respond, and persist when facing mathematical problems. In the context of geometry, geometric habits of mind are particularly relevant because students must identify shapes, compare corresponding sides, and reason about proportional relationships. (Bülbül & Güler, 2023) emphasized that geometric habits of mind play an important role in supporting students' reasoning in geometric problem-solving situations, including tasks that require visual and symbolic coordination.

In mathematics learning that involves mathematical representation, students' learning methods, experiences, and learning environments cannot be separated from their thinking habits (Aringga et al., 2020). These conditions can shape students' mindsets, which begin with habits formed in everyday life. Habits that influence the process of developing individual mathematical abilities are known as habits of mind or thinking habits (Hutajulu & Minarti, 2017). Students with thinking habits at the concrete operational stage still need manipulation and representation of objects to help them connect the knowledge they obtain and apply their representation ability to more complex problems (Hutajulu & Minarti, 2017; Nisa, Mukhlis, & Maswar, 2020; Purwasih, 2018; Suri, Supriadi, Yuliana, & Suherman, 2024). Therefore, habits of mind can help students regulate their learning and solve problems.

Habits of mind are students' thinking habits when they are faced with a problem. These habits can train students to become more productive, critical, creative, persistent, and broad-minded (Tashtoush, 2022). Habits of mind are also considered important characteristics of intelligent thinking in problem-solving and indicators of success in academic, work, and social relationships. In this context, habits of mind can influence the extent to which students can develop and use mathematical representation abilities (Dwirahayu et al., 2018; Tashtoush, 2022). It means that students' representational ability may vary depending on the quality of their thinking habits when interpreting problems, selecting strategies, evaluating answers, and communicating mathematical ideas. Recent studies also support the relationship between habits of mind and students' mathematical representation competence. Ahmad, Akhsani, and Mohamed (2023) reported that students' habits of mind significantly influence their mathematical representation competence at the junior high school level. Similarly, Nauli, Harisman, & Armiati (2024) found that junior high school students with stronger problem-solving abilities showed important habits of mind indicators, including persistence and flexible thinking.

Previous studies have shown that mathematical representation ability is important in helping students solve mathematical problems, communicate mathematical ideas, and understand mathematical concepts (Hasnarika, 2022; Hilmi et al., 2023; Mujib, 2022). Other studies have also emphasized that habits of mind support students in developing mathematical abilities, determining solutions, and regulating their thinking processes (Dwirahayu et al., 2018; Hutajulu & Minarti, 2017; Maarif & Akbari, 2024; Maarif & Fitriani, 2023; Tashtoush, 2022). These findings indicate that mathematical representation and habits of mind are two important constructs in mathematics education. However, previous studies have generally discussed the relationship between mathematical representation and habits of mind in a broad descriptive manner. The existing studies have not sufficiently explained how students' pictorial, symbolic, and verbal representations emerge and develop at more specific levels of habits-of-mind performance.

Based on this condition, there is a research gap that needs to be addressed. Previous studies have not clearly mapped students' mathematical representation trajectories across five levels of performance in the habits of mind, namely beginner, limited, developing, proficient, and superior. In addition, previous studies have not sufficiently demonstrated whether students' representations develop linearly from one level to another or whether there are intersections between levels. For example, students at the developing level may exhibit representational characteristics that are close to those of the proficient level. In contrast, students at the proficient level may still experience weaknesses in verbal explanation or conceptual accuracy. This gap is particularly significant in light of the PISA 2022 findings, which highlight Indonesian students' low confidence in using representational forms such as diagrams and graphs, suggesting that representational deficiencies are a key factor underlying Indonesia's overall low mathematics performance (OECD, 2022). A more detailed map of students' representation trajectories helps teachers better understand students' mathematical thinking and design more targeted interventions.

Therefore, the novelty of this study lies in mapping students' trajectories of mathematical representation across five levels of performance in solving similarity and congruence problems. This study does not merely analyze students' mathematical representation ability based on habits of mind, but also traces how pictorial, symbolic, and verbal representations appear, develop, overlap, and differ at each performance level. Using this trajectory-mapping perspective, this study provides a more detailed account of the characteristics, strengths, weaknesses, and intersections of students' mathematical representations across beginner and advanced levels.

Based on the gap and novelty described above, this study aims to map students' mathematical representation trajectories across five levels of performance in the habits of mind for similarity and congruence problems. Specifically, this study focuses on students' pictorial, symbolic, and verbal representations. The results of this study are expected to contribute to mathematics education by providing a qualitative map of students' representational development based on habits of mind performance, enabling teachers to identify students' representational difficulties and design more appropriate learning support.

Research Methods

The approach used in this study is a descriptive qualitative approach. Qualitative research aims to investigate and explain in detail and systematically the characteristics of certain activities, conditions, or phenomena. In this study, a descriptive qualitative approach was used to map students' trajectories of mathematical representation across five Habits of Mind performance levels in solving similarity and congruence problems. The research focused on three indicators of mathematical representation ability: pictorial, symbolic, and verbal.

The participants in this study were 30 ninth-grade junior high school students who had studied material on similarity and congruence. The participants were selected using purposive sampling because the study required students with sufficient prior learning experience related to the topic under investigation. After the Habits of Mind questionnaire was administered, students were classified into five performance levels, namely beginner, limited, developing, proficient, and superior. Furthermore, one student from each performance level was selected as the main subject for deeper qualitative analysis through written test analysis and semi-structured interviews.

The instruments used in this study consisted of a mathematical representation ability test, a Habits of Mind questionnaire, and a semi-structured interview guide. The mathematical representation ability test was designed to reveal students' pictorial, symbolic, and verbal representations in solving similarity and congruence problems. The Habits of Mind questionnaire was adapted from The Institute for Habits of Mind and was based on the indicators of Habits of Mind developed by Costa and Kallick. The semi-structured interview guide was used to explore students' thinking processes, reasons for choosing certain solution strategies, difficulties in constructing representations, and their ability to explain the solutions written in the test.

Before being used in the research, the mathematical representation ability test, the Habits of Mind questionnaire, and the semi-structured interview guide were validated by three experts: one Professor in mathematics education, one Doctor in mathematics education, and one Master's in mathematics education. The validation was conducted to ensure content validity, construct suitability, language clarity, level of difficulty, suitability with the research objectives, and alignment between the instruments and the indicators of mathematical representation and Habits of Mind performance. The validation of the mathematical representation ability test focused on the suitability of the test items with the indicators of pictorial, symbolic, and verbal representations, the relevance of the items to similarity and congruence material, the clarity of the problem statements, and the suitability of the scoring rubric. The validation of the Habits of Mind questionnaire focused on the suitability of each statement relative to the Habits of Mind indicators, the clarity of the language, and the appropriateness of the response scale. Meanwhile, the validation of the semi-structured interview guide focused on the relevance of the interview questions to the research objectives, their ability to explore students' thinking processes, and their clarity for junior high school students.

The validators provided suggestions for improving the instruments, particularly in terms of wording, clarity of instructions, suitability of mathematical contexts, and consistency between the test items, indicators, and scoring rubric. Based on the validators' suggestions, several revisions were made before the instruments were administered to the students. The revisions included improving the wording of the test items, clarifying the instructions, adjusting the interview questions, and refining the scoring rubric better to reflect the three indicators of mathematical representation ability. After the revision process, the instruments were deemed appropriate for use in the research.

Data were collected through three stages. First, the Habits of Mind questionnaire was administered to classify students into five performance levels. Second, the mathematical representation ability test was given to identify students' pictorial, symbolic, and verbal representations in solving similarity and congruence problems. Third, semi-structured interviews were conducted with selected students at each Habits of Mind performance level to clarify their written responses and deepen their mathematical thinking.

The data were analyzed using qualitative analysis, consisting of data reduction, data display, and conclusion drawing. In the data reduction stage, students' answers were selected, coded, and classified according to indicators of mathematical representation and Habits of Mind performance levels. In the data display stage, the results of students' written work and interview data were presented as descriptions, tables, and representation profiles. In the conclusion-drawing stage, the researcher interpreted the patterns in students' mathematical representation trajectories across the five Habits of Mind performance levels. Data triangulation was used by comparing the results of the mathematical representation ability test, Habits of Mind questionnaire, and interview data to ensure the credibility of the findings.

Results and Discussions

The determination of the level of performance of the habits of Mind above is in accordance with the explanation in the previous chapter about the five levels of performance of habits of Mind. The questionnaire was modified from The Institute for Habits of Mind (2014) and consists of statements based on the habits of mind indicators according to Costa and Kallick (2008). The scale used in this questionnaire is as follows.

Table 1. Habits of Mind Performance Level Score Range

Performance Level	**Beginner**	**Limited**	**Developing**	**Proficient**	**Superior**
Score Range	0 – 3	4 – 9	10 – 18	19 – 24	25 – 28

After administering the Habits of Mind questionnaire, the following results were obtained.

Table 2. Level of Performance of Students' Habits of Mind

Habits of Mind Performance Level	**Many Subjects**
Beginner	3
Limited	4
Developing	11
Proficient	10
Superior	2
Total	30

The students who were selected for further analysis of the results of the mathematical representation ability test will also be interviewed. Thus, five students were selected as described in Table 3 below.

Table 3. Selection of Research Subjects Based on The Level of Performance of Habits of Mind

Habits of Mind Performance Level	Limited	Beginner	Developing	Proficient	Superior
Student Code	R22	R10	R17	R5	R7

Analysis of students' mathematical representation abilities across three indicators, based on five levels of habits of mind performance, is presented in the following Table 4.

Table 4. Indicators of Mathematical Representation Ability

Mathematical Representation	Indicator
Pictorial Representation	Create images or graphics to solve problems
Symbolic Representation	Solve problems by creating mathematical expression models
Verbal representation	Answer questions using words or written text

Representation of Beginner Category Student Answers on R22

Student answers for R22 are shown in Figure 1 below.

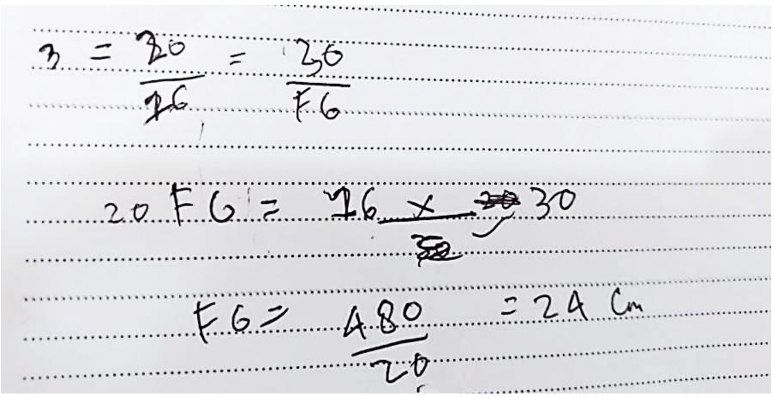

Figure 1. R22 beginner level student answers

In the Pictorial Representation indicator, it is evident that student R22 cannot solve the given problem. Although student R22 provides a picture, it lacks meaning, so it cannot help student R22 solve the problem. From the answer, it can also be seen that student R22 is unable to convey the information in the problem, indicating that student R22 does not understand the problem. Student R22 can already explain the problem, given a little time. The answer given by student R22 is also admitted to be the result of seeing his friend, even though the answer does not help much. One more thing found in the interview results is that student R22 is unable to create a picture to make the problem easier to solve. Student R22 is still confused about what kind of picture to make, and he admits he often cheats his friends because he does not understand the problem the teacher gives. This was also confirmed by the mathematics teacher, who said that student R22 is still weak in basic calculations, especially multiplication and division. Student R22 is also often not focused on the learning process.

In the Symbolic Representation indicator, based on Figure 1. Student R22's answer begins by immediately making a comparison that, if taken at face value, is not relevant to the given question. Although the symbols and mathematical expressions in Student R22's answer appear, their meaning is not yet clear. At the beginning of the comparison on the left side, Student R22 compares 10 to 25, and on the right side, 25 is compared to the letter L. However, in the next step on the right, Student R22 multiplies 6 by 25, whereas in the previous stage, the number 6 did not appear. It indicates that Student R22 does not understand the question or how to solve the comparison problem in it. From the interview results above, it was found that student R22 has not been able to properly interpret questions into mathematical expressions and symbols because student R22 does not understand the questions and lacks in-depth knowledge of similar material. So, student R22 tends to work on questions carelessly, without a strong foundation of knowledge. Student R22 has been able to represent questions as comparisons, even when the direction of the comparison is unclear.

In the Verbal Representation indicator from the image above, it can be seen that student R22 chose only one of the four provided answers, even though three of them used the concept of similarity. The reasons given also lacked meaning. The verbal representation that emerged was still very weak because the reasons given did not address the questions. From the interview results for student R22, it was found that, despite being a beginner at the performance level, student R22 still did not understand the instructions for the questions, resulting in suboptimal answers. The verbal representation in the questions appeared but lacked meaning, and did not answer the questions well, as the interview results showed that student R22 admitted to answering carelessly. He did this because he did not understand the similar material well.

To deepen the analysis of student R22's representational ability, the researcher conducted a semi-structured interview. The following excerpt presents the dialogue between the researcher (R) and student R22 (S):

R : "Can you explain what the problem is asking you to find?"
S : "Honestly, I am not sure, Sir. I did not really understand the problem."
R : "Looking at your answer sheet, why did you start by writing a comparison between 10 and 25?"
S : "I just wrote it down, Sir. I saw something similar in my friend's work, so I tried to follow it."
R : "Did you try to draw a picture to help you understand the problem?"
S : "I tried, but I got confused about what kind of picture to draw. So I just left it."
R : "From the four answer options about similarity, why did you only choose one?"

S : "I chose the one that looked familiar, but I do not really know the reason. I just guessed."
R : "Do you understand the concept of similarity that has been taught in class?"
S : "Not really, Sir. I often get lost when the teacher explains, especially when there is multiplication and division involved."

The interview confirms that R22's representational weakness is rooted in two interconnected issues: a lack of conceptual understanding of similarity and weak habits of mind, particularly persistence and managing impulsivity. This finding is consistent with the mathematics teacher's report that R22 frequently loses focus during lessons and tends to rely on peers rather than engaging with the problem independently. In pictorial representation, R22 was unable to construct a meaningful image because the student did not understand the information contained in the problem. Symbolically, R22 wrote numbers and comparisons by imitating a peer's work, but the representation was not connected to the mathematical structure of similarity. In verbal representation, R22 only guessed the answer and could not provide a conceptual reason. Therefore, R22's mathematical representation trajectory at the beginner level can be described as fragmented, imitative, and not yet meaningful.

Representation of Student Answers in the Limited Category on R10

Student answers R10 can be seen in the following Figure 2.

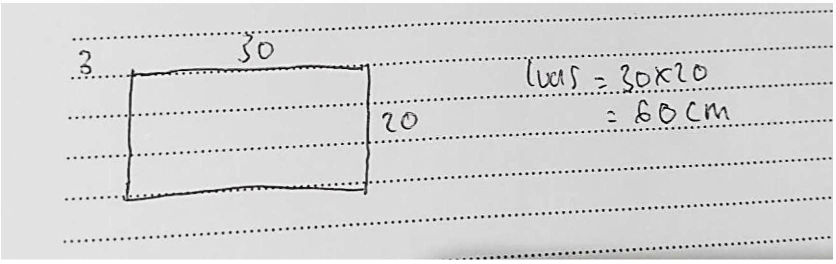

Figure 2. Student R10's Answers in the Limited Category

In the Pictorial Representation indicator, students at the limited habits of mind performance level are already able to represent an image. Although, upon review, the image does not address the given question and cannot help student R10 solve it. From the image, it can also be seen that student R10 has not answered the question according to its purpose. From the interview results above, it was found that student R10 was unable to understand the question well, so the student could not solve it. In addition, student R10 still cannot distinguish between a square and a rectangle, although, when directed, he realized his mistake. The image displayed on the answer sheet is limited to understanding the shape of the cardboard and does not include a photo of it. Solving the question is done in a makeshift way, by subtracting all the numbers listed in the question without considering the correct approach. At a limited level, student R10 still shows a tendency to give up easily and to rely on notes to see examples when working on questions. Based on interview results with mathematics teachers, it was found that R10 students are slow to understand mathematics materials, often working on problems as they are, but their arithmetic skills are quite good. It is just that they still find it difficult to digest questions in story or application form.

In the Symbolic Representation indicator, it is not much different from the beginner-level answer. In the answer of student R10, who has a limited level of habits of mind performance, he is also still unable to optimally represent the problem as a mathematical expression or symbol. The work on the problem also lacks meaning because the final result of solving it is incomplete. In fact, students at a limited level can make comparisons of problems that are not better than those of students at the beginner level. It can be seen in the comparisons made by student R10, which have more meaning than those of students at this limited stage. At a limited level of performance, the ability to use symbolic representation and mathematical expression is still very limited. Even the ability to represent according to the given problem is still below the beginner level; it is just that students' understanding of the problems is better than the beginner level. Problem-solving at a limited level has also not shown the expected ability.

In the Verbal Representation indicator, the results of the analysis of student R10's answers in the picture above can be seen that student R10 chose one answer option, namely enlarging or reducing the photo, the reasons given by student R10 to show verbal ability are also not in accordance with the questions from the questions given. The verbal representation has emerged, but the reasons given do not address the problem.

To clarify the thinking process behind R10's written answer, the researcher conducted a semi-structured interview. The dialogue is presented below.

R : "Can you explain the picture you drew on your answer sheet?"

S : "I drew the cardboard, Sir. The shape is like a rectangle."

R : "Did you include the photo that should be placed on the cardboard?"

S : "Oh, I forgot about the photo. I only drew the cardboard because that was the first thing I read in the problem."

R : "Why did you subtract all the numbers in the problem to find the answer?"

S : "I was not sure how to solve it, so I just subtracted them. I thought maybe that would give the answer."

R : "For the verbal question, you chose enlarging or reducing a photo as an example of similarity. Can you explain why?"

S : "Because when you enlarge a photo, the shape stays the same. However, I am not really sure if my reason is correct."

R : "Do you usually look at your notes when working on math problems?"

S : "Yes, Sir. If I do not see an example first, I do not know how to start."

The interview reveals that R10 has begun to produce representational forms but lacks the conceptual framework to make those representations meaningful. In the pictorial representation, R10 drew the cardboard, but the drawing did not fully represent the situation because the photo was not included. In symbolic representation, R10 used subtraction without a clear proportional relationship, suggesting that the symbolic process remained procedural rather than conceptually grounded. In verbal representation, R10 mentioned photo enlargement or reduction as an example of similarity, but the explanation remained incomplete. The student's dependence on notes and reluctance to attempt unfamiliar steps indicate limited development of habits of mind such as striving for accuracy and persisting. It aligns with the mathematics teacher's observation that R10 has adequate arithmetic skills but struggles to interpret word problems. Thus, R10's mathematical representation trajectory at the limited level can be described as emerging but partial, inconsistent, and weak in mathematical meaning.

Representation of Student Answers in the Developing Category in R17

Student answers in R17 are shown in Figure 3 Here.

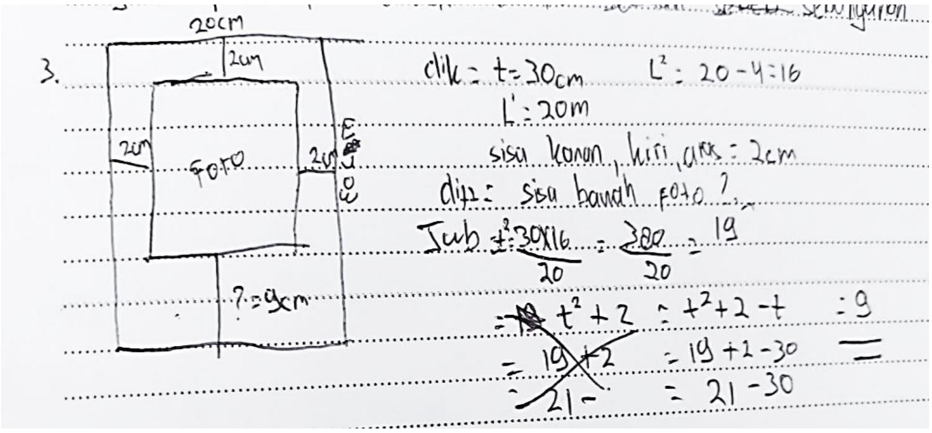

Figure 3. Student R17's Answers in the Developing Category

In the Pictorial Representation indicator, the ability to present questions as images has emerged. The parts of the image have also been written well, starting with the height and width of the cardboard, and the remaining distance between the photo and the right, left, and top cardboard has been described as 2cm. The weakness of student R17 is that the image lacks a shape name written in letters to facilitate comparisons when working on mathematical problems. In mathematical answers, student R17 has shown a systematic approach to questions. It is shown that, before answering the question, student R17 first writes down what is known, what is asked, and the answer. The weakness of student R17 is that the work on the question and the method used to solve it are still incorrect and need significant improvement. From the results of interviews with student R17, several facts emerged: student R17 understood the question. Then student R17 was also able to represent the question as a complete picture, with the elements specified in the question. The next fact is that the level of performance of student R17 has begun to indicate competent performance. According to the math teacher, student R17 is quiet, interacts little with friends, and tends to be alone. Student R17 has good math skills, but he is still not confident in his abilities.

In the Symbolic Representation indicator, the results for student R17's answers in image R17 show that student R17 chose two of the four given answer choices. Then, from each of these choices, student R17 provided good reasons and connected them to the concept of similarity. Student R17's verbal representation ability is good and can even be categorized as competent, but it needs improvement in word order so the reader can understand what has been written. From interviews with student R17, it was found that student R17 could represent questions effectively in verbal form. The reasons given were quite good, although still less precise, and, in terms of conveying reasons and how to speak when interviewed, student R17 was able to explain what he understood in writing and in direct speech. According to the researcher, student R17 already has a competent level of performance.

In the Verbal Representation indicator, the verbal representation ability of student R17 has developed well. Although student R17 chose only one of the four choices, the reasons given by the student have led to the concept of similarity. It shows that his verbal ability is better than that of students at a limited level.

To further explore R17's representational thinking, a semi-structured interview was conducted. The dialogue is as follows.

R : "Why did you start your answer by writing what is known and what is asked?"

S : "I always do that, Sir. It helps me see the problem more clearly before I start calculating."

R : "Your picture is quite detailed, but you did not label the corners with letters. Was there a reason?"

S : "I did not think about it, Sir. I usually draw the shape and write the measurements. Next time I will try to label the corners."

R : "Looking at your calculation, the final answer is not correct. Can you explain how you arrived at it?"

S : "I think I made a mistake when I combined the formulas. I was trying to use the area, but I got confused with the proportion."

R : "For the verbal question, you chose two options. Can you explain why those options represent similarity?"

S : "Because both of them have the same shape but different sizes, like a map and the real place, and a photo that is enlarged."

R : "How confident are you with your understanding of similarity?"

S : "I think I understand it, but sometimes I doubt myself when I am working on the problem."

The interview indicates that R17 has developed a meaningful conceptual understanding of similarity and is able to articulate the underlying reasoning verbally. In the pictorial representation, R17 constructed a relatively complete image by including important information from the problem, although the figure was not labeled with letters. In symbolic representation, R17 attempted to use comparison and area-related reasoning, but errors still appeared in connecting proportion and calculation. In verbal representation, R17 was able to explain similarity through examples of the same shape with different sizes. The systematic working pattern and willingness to reflect on errors suggest that habits of mind such as thinking about thinking and striving for accuracy are beginning to emerge. However, the student's lack of confidence remains a barrier to advancing to a higher performance level. Therefore, R17's mathematical representation trajectory at the developing level can be categorized as meaningful and emerging, but still requiring conceptual refinement.

Representation of Student Answers in the Proficient Category at R5

Student R5's answers are shown in Figure 4 Below.

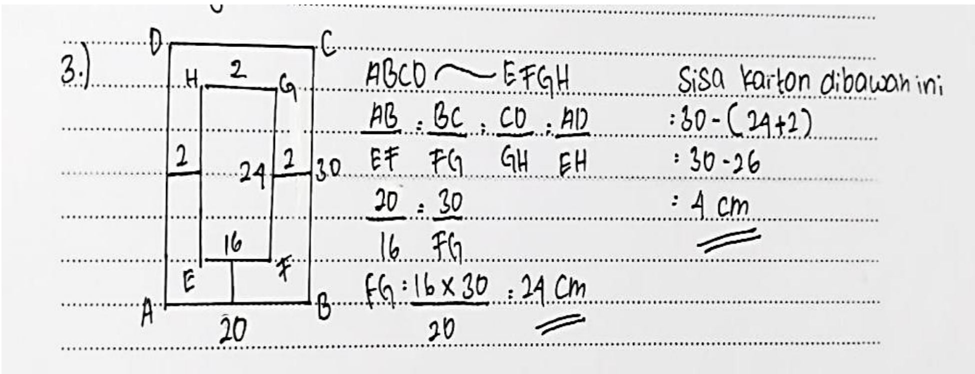

Figure 4. Student R5's Answers in the Proficient Category

In the Pictorial Representation indicator, it can be seen that R5 students have also been able to represent questions in picture form properly and correctly. R5 students have also provided complete and detailed information from the picture, including the size and the naming of flat shapes, or in this case, rectangles. R5 students have also been able to work on questions using the pictures they made, making it easier for them to solve the questions properly. The weakness of R5 students is that their work on the questions is less systematic. Because R5 students do not first explain the known things and the questions given. It shows that students at the proficient performance level share the same characteristics: they are less systematic in their approach to questions and tend to want to finish quickly. From the interview results, it was found that R5 students had indeed understood the intent and purpose of the questions. Then R5 students have also realized that picture representation is very necessary to solve the questions given. However, R5 students made a small mistake: they did not draw a line between the cardboard and the top photo, so R5 students' pictures appear to make the length of the GH side 2 cm. It was only realized during the interview.

In the Symbolic Representation indicator, R5 students understand the questions well and know how to solve the given ones. Representation with mathematical expressions has also appeared, including comparisons, the introduction of variables to represent quantities not yet known, cross-multiplication, and the formula for the area of a rectangle. It is just that R5 students do not express the area units of a rectangle clearly and do not simplify the final comparison in their answers.

In the Verbal Representation indicator, it was found that student R5 did not fully understand the concept of similarity. His verbal representation ability was good, but the reasons given were still not quite right. Because student R5 considered that similarity is all flat shapes that have the same shape but different sizes. Student R5 forgot that the size requirement for similarity must have a comparison that is equal or the same.

To explore the thinking process underlying R5's answer, the researcher conducted a semi-structured interview. The excerpt is presented below:

R : "Your picture is well-drawn with the corners labeled ABCD and EFGH. Can you explain why labeling is important?"

S : "Labeling makes it easier to identify the corresponding sides when comparing the two rectangles. Without labels, I would get confused."

R : "You went straight into calculations without first writing what is known and what is asked. Was there a reason?"

S : "I wanted to finish quickly, Sir. I already understood the problem in my head, so I did not write it down."

R : "Looking at your drawing, the GH side appears to be 2 cm. Was that intentional?"

S : "Oh, now I see the mistake. I forgot to draw the distance between the top of the photo and the cardboard. The GH side should actually be longer."

R: "Your verbal explanation states that similarity means shapes have the same form but different sizes. Is that the complete definition?"

S : "I think so, Sir. As long as the shape is the same and the size is different, it is similar."

R : "What about the proportion of the corresponding sides? Is that also required?"

S : "Oh yes, the sides should also be proportional. I forgot to mention that part."

The interview reveals that R5 demonstrates strong procedural fluency and metacognitive awareness, as evidenced by the student's ability to recognize errors when prompted. In pictorial representation, R5 was able to construct a clear image and label the figure to identify corresponding sides. However, a minor error appeared because the distance between the upper part of the photo and the cardboard was not drawn. Symbolically, R5 used comparison, variables, cross-multiplication, and the area formula, though the solution was not fully systematic. In verbal representation, R5 was able to explain similarity generally, but the definition was incomplete because the student initially did not mention the proportionality of corresponding sides. The tendency to rush through problems also indicates that habits of mind such as striving for accuracy and checking for understanding still require refinement. Thus, R5's mathematical representation trajectory at the proficient level is structured and meaningful but still requires refinement in conceptual accuracy and written explanation.

Representation of Student Answers in the Superior Category in R7

Student R7's answers can be seen in Figure 5 Below.

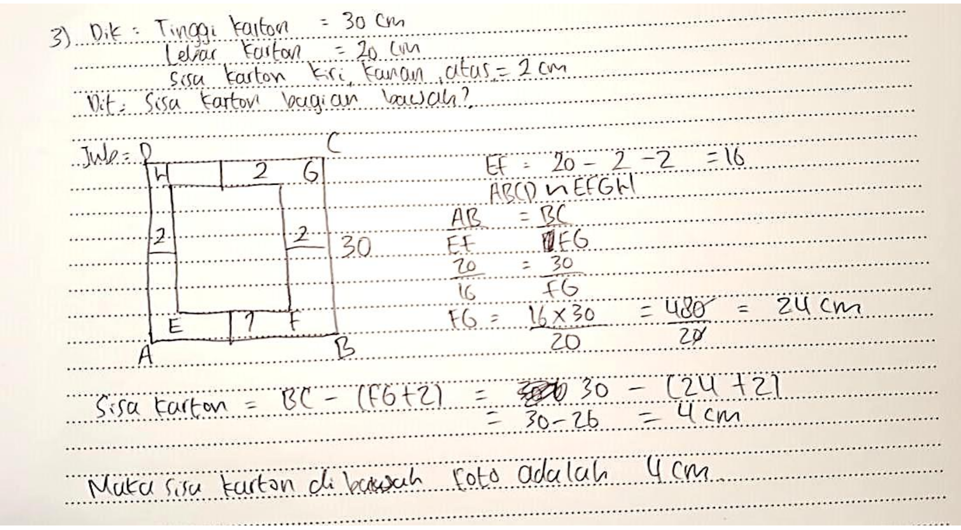

Figure 5. Student Answers in the Superior Category R7

In the Pictorial Representation indicator, it can be seen that R7 students have also been able to represent questions in image form well. R7 students have also been able to make complete and clear images of the size and naming of the image, although the images are still not neat and can still be understood. The work on mathematical problems has also been well structured: start by writing down what is known and what is asked in the problem, then translate them into images, and continue with mathematical calculations. From the interview results, it was found that superior students already have strong image representation skills. R7 students said that creating images would make it easier to solve similarity problems by showing which sides correspond, so there are no mistakes when working on the problem. It is a characteristic of students with a superior level of habits of mind performance.

In the Symbolic Representation indicator, overall, R7 students are very good at solving this problem. The work begins by recording what is known, writing down questions, and solving problems very well and systematically. The mathematical representation that appears is not only a mathematical expression; the verbal representation ability in R7 students' answers has also emerged very well. The representation of mathematical expressions appears in accordance with the intent and purpose of the problem, the comparison of length and width, the example of the unknown with a variable, the formula for the area of a rectangle complete with the correct units and the comparison of the area of the photo before and after being enlarged are all presented in the form of appropriate mathematical expressions.

In the Verbal Representation indicator, it can be seen that student R7 chose three of the four options given: maps and scales, photo enlargement/reduction, and making object models. For his verbal representation ability, it can be seen from the reasons given by student R7, who stated that the three options have the same shape but different sizes. In addition, student R7 wrote that, in this case, the fourth option, tiling, is an example of congruence with the same shape and size.

To confirm the depth of R7's representational thinking, the researcher conducted a semi-structured interview. The dialogue is presented below:

R : “Your answer begins by writing what is known and what is asked. Why is that important to you?”

S : “It helps me organize my thinking, Sir. When I write down the known and asked information, I can clearly see what I need to find and which formulas to use.”

R : “Why did you draw a picture before doing the calculations?”

S : Drawing helps me see which sides correspond to each other. In similarity problems, identifying the corresponding sides is the most important step, and a picture makes that visible.”

R : “You used variables and the area formula in your symbolic representation. How did you decide to use them?”

S : “Because some values were unknown, I represented them with variables. Then I used the area formula to compare the photo and the cardboard.”

R : “For the verbal question, you chose three options as examples of similarity. Can you explain your reasoning?”

S : "Maps and scales, photo enlargement, and object models all have the same shape but with proportional differences in size. The fourth option, tiling, is congruent because the shapes are exactly the same in both shape and size."

R : "How do you usually approach a difficult mathematics problem?"

S : "I read the problem carefully, then I try to draw it. If the drawing is clear, the solution usually becomes easier. If I get stuck, I try another strategy until it works."

The interview demonstrates that R7 has fully integrated pictorial, symbolic, and verbal representations into a coherent problem-solving strategy. In the pictorial representation, R7 used the image to accurately identify the corresponding sides. In symbolic representation, R7 used variables, proportional relationships, and the area formula appropriately. In verbal representation, R7 distinguished between similarity and congruence, explaining that similarity involves the same shape with proportional size differences, while congruence involves the same shape and the same size. The student's ability to distinguish between similarity and congruence verbally, combined with systematic written work and reflective thinking, indicates that habits of mind such as thinking flexibly, striving for accuracy, applying past knowledge to new situations, and metacognition operate at an advanced level. Therefore, R7's mathematical representation trajectory at the superior level can be categorized as complete, systematic, flexible, and conceptually meaningful.

After analyzing data on ten students based on the mathematical representation ability indicator at each level of habits of mind performance, a recapitulation of students' mathematical representation abilities was obtained from each level of habits of mind as follows.

Table 5. Students' Mathematical Representation Ability at the Habits of Mind Performance Level

Habits of mind performance level	Mathematical representation ability indicators			Information
	Pictorial Representation	Symbol Representation	Verbal Representation	
Beginner (R22)	There isn't any, because I don't understand the question.	It's still very simple and already includes a description.	Verbal abilities are still very limited.	Students at the beginner level have better symbol abilities than other indicators.
Limited (R10)	The image is still simple and meaningless	Symbols appear without any clear explanation	Verbal abilities are limited and inadequate	Students with limited levels have better verbal abilities than other indicators.
Development (R17)	The problem can be represented in the form of a picture very well, but the solution to the	Symbolic abilities emerge well and can solve the given problems.	Verbal representation in writing is less clear but can be clarified in direct (oral) communication.	R17 students already have representation skills equivalent to the proficient level

Habits of mind performance level	Mathematical representation ability indicators			Information
	Pictorial Representation	Symbol Representation	Verbal Representation	
	problem is still not quite right.			
Proficient (R5)	The image representation is good and loads the image elements well, but there are still minor errors.	Symbolic abilities are good, but not well structured	Verbal representation in written form is less clear but can be clarified in direct (oral) communication.	Students' mathematical representation abilities at a proficient level are associated with greater symbolic indicators than other indicators.
Superior (R7)	Representing the problem in the form of a picture very well and in detail.	The representation of mathematical symbols and expressions is structured according to mathematical principles and supports metacognitive thinking.	Written and oral representations are adequate and clear.	Students at the superior level have the same ability to represent mathematics. All three indicators appear at the same level of ability.

From the results of the analysis above, and based on the average score of students' mathematical representation ability, as shown in Table 6, the following conclusions are drawn:

Table 6. Average score of students' representation ability test

Question Number	Mathematical Representation Ability Indicators	Score	
		Average	%
1	Symbolic Representation	2,17	72%
2	Verbal Representation	1,60	53%
3	Pictorial Representation	1,87	62%

Thus, it can be concluded that students' overall mathematical representation ability is best represented by the symbolic representation indicator, followed by the image representation, and lastly the verbal representation ability. Students' symbolic representation ability is better than other indicators. Visual (image) and verbal representations are at a higher level than symbolic representations. Meanwhile, differences in students' mathematical representation ability across indicators are caused by inaccuracies in reading and solving questions. It indicates that students' mathematical representation ability for each indicator can vary across research variables and other factors.

Table 7. Students' Mathematical Representation Ability Reviewed from Habits of Mind

Mathematical representation ability indicators	Habits of mind performance level	
Pictorial Representation	• Not understanding the question well, so the image representation does not appear. • Giving up easily so as not to complete the question • Choosing a shortcut by cheating on friends	Beginner (R22)
Symbol Representation	• Symbol/expression representation appears, but does not match the given question. • Does not understand the similarity material, so the question is not solved • Gives up easily, so he does the question carelessly	
Verbal Representation	• Verbal representation, in both oral and written form, is inadequate.	
Pictorial Representation	• The image representation has appeared, but is still simple and has no meaningful value. • The questions are solved with little understanding, so the results are not optimal.	**Limited (R10)**
Symbol Representation	• Symbol representation appears but is less relevant to the given question. • Identifying less structured questions Working on questions hesitantly and doubtfully because of not being sure of one's own abilities	
Verbal Representation	• Verbal representation in oral and written form is inadequate.	
Pictorial Representation	• Already able to create a picture representation, but there are still some basic errors, so it does not help much in solving the problem. • Working on the problem has started to be systematic and focused, but it often loses focus, so that the final result is also not optimal. • Still trying to solve the problem	Developing (R17)
Symbol Representation	• Symbol representation has appeared and is relevant to the problem, but there are still some errors. • Often uses the same method to solve problems due to limited knowledge. • Makes several errors in solving problems	
Verbal Representation	• Verbal representation in written form is unclear, but cannot be clarified in direct (oral) communication.	

Mathematical representation ability indicators	Habits of mind performance level	
Pictorial Representation	• Able to present questions in the form of images well, only there are some minor errors in the depiction presented • Questions can be solved with the final result of the correct answer. • The work on the questions seems rushed and not systematic	Proficient (R5)
Symbol Representation	• Able to present problems in the form of symbols and mathematical expressions but still incomplete. • Implementing a solution strategy in a systematic way • Able to think metacognitively in oral responses to the given problem solving	
Verbal Representation	• Written verbal representation is less clear, but can be clarified through direct (oral) communication and can be directed to clarify written communication	
Pictorial Representation	• Presenting in the form of images very well and in detail • Being able to develop the knowledge possessed to solve problems • Indicators of representation ability in the form of images, mathematical and verbal expressions appear in solving the answer	Superior (R7)
Symbol Representation	• Symbol representation and mathematical expressions appear according to mathematical structures. • Identifying problems in a structured manner • Being able to think metacognitively in oral and written responses to solving problems given • Being able to implement strategies in a systematic way but can be developed to find new strategies.	
Verbal Representation	• Written and oral verbal representation is adequate and clear	

Discussion

The findings of this study reveal that students' abilities to represent mathematically in solving similarity and congruence problems vary systematically across five Habits of Mind performance levels. The diversity of representational strategies observed during the test is closely related to students' Habits of Mind, which directly influence the way they identify information, select solution strategies, and communicate mathematical ideas. This finding strengthens the results of (Cuoco, A. L., Goldenberg, E. P., & Mark, 1996; Lim, K. H., & Selden, 2009), who emphasized that Habits of Mind play a foundational role in shaping students' mastery of mathematical concepts. The present study extends their work by demonstrating that this influence is not uniform but manifests differently at each performance level, producing distinct representational trajectories rather than a single linear pathway. This finding also strengthens Anggraena (2019); Annisa and Liberna (2025), who argued that students' identification of data, facts, and strategies triggers the generalization process that ultimately produces pictorial, symbolic, and verbal representations. This finding also aligans with Ahmad, Akhsani, and Mohamed (2023), who reported that students' habits of mind significantly influence their mathematical representation competence at the junior high school level. Therefore, the present study extends previous findings by showing that this influence appears differently across five Habits of Mind performance levels.

At the beginner level, student R22 produced fragmented representations that lacked meaningful connections to the given problem. The interview confirmed that R22 did not understand the material on similarity and worked carelessly, often copying from peers. This finding is consistent with Barokah and Madawistama (2024), who observed that students with weak Habits of Mind tend to act impulsively, ignoring logical and reflective thinking. However, the present study contrasts with the more general claim in Rahmah and Zulfadewina (2025) that beginner-level students exhibit only "very low representation ability" without further specification. The present study provides a finer-grained description: although R22's pictorial representation lacked meaning, the symbolic indicator still emerged in a rudimentary form (comparison expressions), suggesting that, even at the beginner level, symbolic representation may appear earlier than pictorial representation when students attempt to imitate procedural patterns from peers. This nuance has not been reported in prior descriptive studies.

At a limited level, student R10 began producing pictorial representations, but the drawings did not capture the essential information of the problem. This finding aligns with Hasnarika (2022); Hilmi et al. (2023), who reported that students with limited thinking habits often produce representations that are form-based but lack conceptual depth. Unlike Hilmi et al. (2023), who focused mainly on the presence or absence of representation, the present study reveals an unexpected pattern: students at the limited level demonstrated slightly better verbal representation than those at the beginner level, but their symbolic representation was weaker. It contradicts the linear assumption in earlier descriptive research that all three representational indicators develop simultaneously across performance levels. The present study suggests that representational development is asynchronous; different indicators may grow at different rates within the same student.

At the developing level, student R17 demonstrated the ability to construct systematic pictorial representations and to connect verbal reasoning with the concept of similarity. This finding strengthens Dwirahayu et al. (2018), who concluded that Habits of Mind influence students' internal representation processes. However, the most distinctive finding of this study lies at this level: student R17 displayed representational characteristics that overlapped with those at the proficient level, particularly in symbolic and verbal indicators. This finding contrasts with the linear progression assumption implied in Janah et al. (2022); Aini and Hidayah (2024), who suggested that higher Habits of Mind directly correspond to higher representation ability in a strictly proportional manner. The present study demonstrates that the boundary between developing and proficient levels is permeable, and that students may exhibit characteristics of an adjacent level even before fully completing their current stage. This non-linear trajectory is a novel contribution that has not been explicitly documented in previous research on mathematical representation and Habits of Mind.

At the proficient level, student R5 demonstrated strong symbolic representation and well-organized pictorial representation but exhibited minor weaknesses in systematic written work and in fully defining similarity. This finding aligns with

Hutajulu and Minarti (2017), who argued that proficient students demonstrate metacognitive awareness and intrinsic motivation. However, it also partially contrasts with their claim that proficient students consistently strive for accuracy. The present study found that R5, despite operating at the proficient level, tended to rush through problems and only recognized errors when prompted during the interview. It suggests that the proficient-level trajectory is characterized by emerging metacognition that is not yet fully self-regulated, a nuance not previously captured in the literature.

At the superior level, student R7 coordinated pictorial, symbolic, and verbal representations flexibly and meaningfully, accurately distinguishing similarity from congruence. This finding strengthens (Maarif & Fitriani, 2023; Siagian et al., 2022; Tashtoush, 2022), who argued that superior Habits of Mind enable students to engage in productive, critical, and reflective mathematical thinking. The interview with R7 confirmed the integration of multiple Habits of Mind, including thinking flexibly, striving for accuracy, applying past knowledge, and metacognition, fully consistent with Costa and Kallick's framework, as referenced by Aini and Hidayah (2024). Unlike the descriptive findings of Aringga et al. (2020), which characterized superior students primarily in terms of cognitive style, the present study demonstrates that superior-level representational ability is also shaped by the student's capacity to strategically switch between representations, a behavior that emerges only when multiple Habits of Mind operate simultaneously. This finding is also supported by Bülbül (2021), who reported that beliefs about geometric habits of mind are positively related to self-efficacy in geometry. This supports the present finding that superior-level students demonstrate confident, integrated, and flexible representational strategies when solving similarity and congruence problems.

A cross-cutting finding of this study is that representational development does not always progress linearly across the five Habits of Mind performance levels. It contrasts with the proportional relationship reported by Ikhsan, Dewi, and Waluya (2024); Janah et al. (2022); Nisa, Mukhlis, and Maswar (2020). The findings of this study are also consistent with Nauli, Harisman, and Armiati (2024), who observed that junior high school students with strong problem-solving abilities exhibited several indicators of habits of mind, including persistence and thinking flexibility. However, unlike Nauli, Harisman, and Armiati (2024), who categorized students into three levels, the present study extends the framework to five performance levels and identifies non-linear trajectories between them. The present study identifies clear intersections, particularly between the developing and proficient levels, and between the limited and beginner levels, in terms of which representational indicator emerges first. These intersections suggest that the categorization of Habits of Mind performance, although useful as a diagnostic framework, should be interpreted as a continuum rather than as discrete stages. This is consistent with the cautionary note in Cuoco, Goldenberg, and Mark (1996) that the development of mathematical habits requires longitudinal observation and may not be fully captured by single-session assessments.

The findings also align with the broader concern reflected in the PISA 2022 results (OECD, 2022), which identified that Indonesian students struggle particularly with extracting mathematical information from diagrams, graphs, and other representational forms. This study strengthens the international observation at the classroom level by showing that students' representational weaknesses are systematically related to the quality of their Habits of Mind. This finding adds evidence to the argument by Malasari, Herman, and Jupri (2019) and Hasnarika (2022) that representational ability should be developed alongside thinking habits rather than treated as an isolated skill.

Practical Implications for Teaching

The trajectory map produced by this study offers several practical implications for mathematics teachers, particularly at the junior high school level: (1) teachers can use the five-level trajectory as a diagnostic tool to identify which representational indicator (pictorial, symbolic, or verbal) is the weakest in a particular student and design targeted scaffolding accordingly. For example, beginner-level students may benefit more from guided visualization activities, while developing-level students may need support with verbal articulation rather than with basic procedures; (2) because representational development is asynchronous and non-linear, teachers should avoid grouping students rigidly based on a single performance score. Instead, a mixed grouping that allows developing-level students to interact with proficient-level peers may accelerate the transition at the intersection of the two levels, as suggested by Coşkun, Özen-Ünal, Eryalçın, Yazıcı, and Ünal (2023) regarding thinking interdependently; (3) teachers should explicitly model Habits of Mind during instruction, particularly persisting, striving for accuracy, and metacognition, by thinking aloud while solving similarity and congruence problems. This explicit modeling addresses the impulsivity and lack of reflection observed at the beginner and limited levels in this study; (4) the assessment of mathematical representation ability should not rely solely on written tests. The interview data in this study revealed that some students (particularly at the proficient level) possessed deeper conceptual understanding than their written work suggested. Teachers may therefore incorporate brief oral clarifications or think-aloud sessions into formative assessment; and (5) given the low confidence of Indonesian students in working with diagrams and graphs reported in PISA 2022 (OECD, 2022), teachers are encouraged to prioritize pictorial representation as an entry point in similarity and congruence instruction, since this is the representational form most directly linked to the geometric reasoning required by the topic.

Conclusions and Suggestions

Based on the research aim to map students' mathematical representation trajectories across five Habits of Mind performance levels in solving similarity and congruence problems, this study concludes that students' pictorial, symbolic, and verbal representations develop through five distinct trajectories. These trajectories are not merely categories of ability, but describe how students construct, connect, and communicate mathematical ideas at each level of Habits of Mind performance.

First, the beginner trajectory is characterized by fragmented and imitative representations. Students at this level have difficulty understanding the problem, constructing meaningful pictorial representations, and connecting mathematical symbols to the concept of similarity. Although symbolic forms may appear, they are not grounded in a clear conceptual understanding. Verbal representation is also very limited because students tend to guess answers without providing relevant mathematical reasons. This indicates that beginner-level students still need strong guidance in identifying information, understanding the problem context, and constructing initial representations.

Second, the limited trajectory shows emerging but partial representation ability. Students at this level begin to produce pictorial or verbal representations, but these remain incomplete, inconsistent, and weak in mathematical meaning. Their symbolic representation is generally procedural and not yet connected to proportional reasoning. In similarity and congruence problems, lower-level students may recognize familiar contexts, such as photo enlargement or reduction, but they still struggle to explain the underlying mathematical relationship. This trajectory indicates that students at the limited level require scaffolding to connect visual information, symbolic expressions, and verbal explanations.

Third, the developing trajectory demonstrates meaningful but not yet stable representational development. Students at this level can construct more relevant pictorial representations and begin to connect them with symbolic and verbal reasoning. However, errors may still occur in proportional reasoning, calculation, or the systematic use of labels and mathematical structures. An important finding at this level is that developing students may exhibit characteristics close to the proficient level, especially in verbal reasoning and in the ability to explain mathematical ideas orally. This shows that the transition from developing to proficient is not rigid, but contains an intersection between levels.

Fourth, the proficient trajectory is characterized by structured and meaningful representations, especially in pictorial and symbolic forms. Students at this level can identify corresponding sides, use variables, apply comparison, and perform mathematical procedures more appropriately. However, the trajectory is not yet fully complete, as students may still show weaknesses in systematic written work, accuracy, and the verbal explanation of concepts. For example, students may understand similarity procedurally but still provide an incomplete definition by failing to mention the proportionality of corresponding sides clearly. This indicates that proficient-level students need support in strengthening conceptual accuracy and written mathematical communication.

Fifth, the superior trajectory exhibits complete, systematic, flexible, and conceptually meaningful representation ability. Students at this level can coordinate pictorial, symbolic, and verbal representations coherently. They can construct meaningful images, identify corresponding sides, use variables and proportional relationships appropriately, and distinguish similarity from congruence through clear verbal explanations. Students at the superior level also demonstrate stronger Habits of Mind, such as thinking flexibly, striving for accuracy, applying prior knowledge to new situations, and engaging in metacognition. Therefore, the superior trajectory represents the most integrated form of mathematical representation ability in this study.

A key finding of this study is that students' mathematical representation trajectories do not always progress linearly across the five Habits of Mind performance levels. The results show that there are intersections between levels, particularly between the developing and proficient levels. A student at the developing level may demonstrate representational characteristics close to the proficient level. In contrast, a student at the proficient level may still show weaknesses in verbal explanation or conceptual precision. This finding indicates that Habits of Mind performance levels should be understood as a continuum rather than as completely separate categories.

The practical implication of this study is that mathematics teachers can use the five-level trajectory map as a diagnostic tool to identify students' representational strengths and weaknesses. Beginner and limited-level students need support in understanding the problem context and constructing basic pictorial representations. Developing-level students need guidance in connecting pictorial, symbolic, and verbal forms more accurately. Proficient-level students need support in improving written explanation, conceptual precision, and accuracy. Superior-level students can be given more complex tasks that require flexible movement across different representations.

For future research, it is recommended that similar studies involve a larger number of participants and different mathematical topics to examine whether the same trajectory pattern holds across contexts. Longitudinal research is also needed to observe how students move from one Habits of Mind performance level to another over time. In addition, future studies may investigate the effects of specific learning interventions, such as problem-based learning, visual scaffolding, or metacognitive instruction, on students' trajectories of mathematical representation.

Future research may also examine students' mathematical representation trajectories by integrating geometric habits of mind, students' self-efficacy, and mathematical modeling ability. This direction is relevant because Bülbül (2021) emphasized the role of beliefs in geometric habits of mind, while Aprilia, Asih, Usdiyana (2023) showed that mathematical habits of mind are related to students' mathematical modeling ability.

Acknowledgements

We would like to thank all participants, educators, and researchers who contributed to the successful implementation of the research on the analysis of students' representative abilities in terms of habits of mind.

References

Ahmad, A., Akhsani, L., & Mohamed, Z. (2023). The profile of students' mathematical representation competence, self-confidence, and habits of mind through problem-based learning models. *Infinity Journal*, *12*(2), 323–338. https://doi.org/10.22460/infinity.v12i2.p323-338

Aini, K. N., & Hidayah, N. (2024). Analisis kemampuan representasi matematis siswa dalam menyelesaikan soal matematika ditinjau dari tipe kepribadian florence littauer [Analysis of students' mathematical representation abilities in solving mathematical problems viewed from Florence Littauer's personality type]. *Indo-MathEdu Intellectuals Journal*, *5*(2), 1916–1922. https://doi.org/10.54373/imeij.v5i2.987

Anggraena, Y. (2019). Pengembangan kurikulum matematika untuk meningkatkan kemampuan siswa dalam penalaran dan pemecahan masalah [Developing a mathematics curriculum to improve students' reasoning and problem-solving abilities]. *Alifmatika: Jurnal Pendidikan Dan Pembelajaran Matematika*, *1*(1), 15–27. https://doi.org/10.35316/alifmatika.2019.v1i1.15-27

Annisa, G., & Liberna, H. (2025). Peran disposisi matematis dan kemandirian belajar dalam meningkatkan kompetensi representasi matematis: Kajian literatur [The role of mathematical disposition and learning independence in improving mathematical representation competence: A literature review]. *JIIP (Jurnal Ilmiah Ilmu*

Pendidikan), *8*(2), 1169–1175. http://jiip.stkipyapisdompu.ac.id

Aprilia, A., Asih, E. C. M., & Usdiyana, D. (2023). The effect of mathematical habits of mind and early mathematical ability on modeling ability of high school students. *JTAM (Jurnal Teori Dan Aplikasi Matematika)*, *7*(4), 914–923. https://doi.org/10.31764/jtam.v7i4.11924

Aringga, D., Shodiqin, A., & Albab, I. U. (2020). Penelusuran kebiasaan berpikir (habits of mind) matematis siswa dalam menyelesaikan soal cerita bilangan pecahan ditinjau dari gaya kognitif [Exploring students' mathematical thinking habits (habits of mind) in solving fractional story problems from the perspective of cognitive style.]. *Thinking Skills and Creativity Journal*, *2*(2), 121–129. https://doi.org/10.23887/tscj.v2i2.22666

Barokah, R., & Madawistama, S. T. (2024). Kemampuan literasi matematis dalam menyelesaikan soal AKM ditinjau dari keyakinan matematis siswa [Mathematical literacy ability in solving AKM problems is reviewed from students' mathematical beliefs.]. *Jurnal Kongruen,* *3*(3), 208-217. https://jurnal.unsil.ac.id/index.php/kongruen/article/view/14085

Bülbül, B. Ö., & Güler, M. (2023). Examining the effect of dynamic geometry software on supporting geometric habits of mind: A qualitative inquiry. *Industry and Higher Education*, *37*(1), 100–113. https://doi.org/10.1177/20427530221107776

Bülbül, B. Ö. (2021). Factors affecting prospective mathematics teachers’ beliefs about geometric habits of mind. *Journal of Pedagogical Research*, *5*(2), 36–48. https://doi.org/10.33902/JPR.2021068370

Choiriyaza, A. E., & Fatma, M. (2021). Pemodelan matematika: Dapatkah autograph meningkatkan representasi semiotik matematik siswa? [Mathematical modeling: Can autographs improve students' mathematical semiotic representations?]. *JNPM (Jurnal Nasional Pendidikan Matematika)*, *5*(2), 264–276. https://doi.org/doi.org/10.33603/jnpm.v5i2.1210

Coşkun, İ., Özen-Ünal, D., Eryalçın, S., & Yazıcı, E., & Ünal, D. Ö. (2023). Investigation of pre-service middle school mathematics teachers habits of mind in pattern generalization problems. *Osmangazi Journal of Educational Research*, *10*(2), 52–75. https://doi.org/10.59409/ojer.1359752

Cuoco, A. L., Goldenberg, E. P., & Mark, J. (1996). Habits of mind: An organizing principle for mathematics curricula. *Journal of Mathematical Behavior*, *15*(4), 375–402.

Dwirahayu, G., Kustiawati, D., & Bidari, I. (2018). Pengaruh habits of mind terhadap kemampuan generalisasi matematis [The influence of habits of mind on mathematical generalization ability]. *Jurnal Penelitian dan Pembelajaran Matematika, 11*(2), 91-104. https://doi.org/10.30870/jppm.v11i2.3757

Haryadi, J., Mujib, A., & Siagian, S. S. (2024). Analysis of student image concepts in constructing proof and mathematical communication in terms of gender. *International Journal of Multidisciplinary and Current Educational Research*, *6*(4), 103–109. www.ijmcer.com

Hasnarika. (2022). Penerapan Metaphorical thinking untuk meningkatkan kemampuan representasi matematis dan habits of mind siswa SMP [The application of metaphorical thinking to improve the mathematical representation skills and habits of mind of junior high school students]. *Jurnal Pendidikan MIPA*, *12*, 255–264. https://doi.org/10.37630/jpm.v12i2.586

Hilmi, Y., Matematika, P. P., & Tasikmalaya, U. C. (2023). Analisis Kemampuan Representasi Matematis Siswa SMP Ditinjau dari Habits of Mind. *J-PiMat: Jurnal Pendidikan Matematika,* *5*(2), 979-988. https://doi.org/10.31932/j-pimat.v5i2.2906

Hutajulu, M., & Minarti, E. D. (2017). Meningkatkan kemampuan advanced mathematical thinking dan habits of mind mahasiswa melalui pendekatan keterampilan metakognitif [Improving students' advanced mathematical thinking skills and habits of mind through a metacognitive skills approach]. *JES-MAT*, *3*(2), 177–194. https://doi.org/10.25134/jes-mat.v3i2.690

Ikhsan, N. M., Dewi, N. R., & Waluya, S. B. (2024). Kemampuan representasi matematis siswa pada model pembelajaran preprospec berbantuan aplikasi game android: Systematic literature review (SLR) [Students' mathematical representation ability in the preprospec learning model assisted by an Android game application: Systematic literature review (SLR)]. *PRISMA, Prosiding Seminar Nasional Matematika*, *7*, 253–260. https://proceedings.unnes.ac.id/prisma/article/view/2960

Ikhsan, R., Abdillah, N., Zahari, C., & Wardani, H. (2024). Meta analisis pembelajaran bidang matematika dengan menggunakan model pembelajaran problem based learning [Meta-analysis of mathematics learning using the problem-based learning model]. *Jurnal Cendekia: Jurnal Pendidikan Matematika*, *8*(2), 1409–1417. https://doi.org/10.31004/cendekia.v8i2.3080

Janah, N., Akbar, R. O., & Kusmanto, H. (2022). Analisis kemampuan literasi matematika ditinjau dari habits of mind siswa di SMPN 14 Kota Cirebon [Analysis of mathematical literacy skills reviewed from the habits of mind of students at SMPN 14 Cirebon City]. *Polynom : Journal in Mathematics Education*, *2*(3), 167-180. https://doi.org/10.14421/polynom.2022.23.167-180

Lim, K. H., & Selden, A. (2009). Mathematical habits of mind. *PME-NA Working Group Paper*, *5*(2) 1576-1583. https://scholarworks.utep.edu/math_papers/242/

Lubis, L. K., Lubis, A. M., Fitriani, F., & ... (2024). Analysis of Students' Mathematical Concept Understanding Ability in Terms of Student Resilience at Ban Kadeng School Thailand. *Kreano, Jurnal Matematika Kreatif-Inovatif,* *15*(2), 409-422. https://doi.org/10.15294/bc59ec96

Maarif, S., & Akbari, H. A. (2024). Sociomathematical norm, mathematical resilience and mathematical representation ability in mathematics learning. *Jurnal Penelitian dan Pengembangan Pendidikan,* *8*(3), 621-631. https://doi.org/10.23887/jppp.v8i3.65238

Maarif, S., & Fitriani, N. (2023). Mathematical resilience, habits of mind, and sociomathematical norms by senior high school students in learning mathematics: a structured equation model. *Infinity Journal*, *12*(1), 117–132. https://doi.org/10.22460/infinity.v12i1.p117-132

Malasari, P. N., Herman, T., & Jupri, A. (2019). Kontribusi habits of mind terhadap kemampuan literasi matematis siswa pada materi geometri [The contribution of habits of mind to students' mathematical literacy skills in geometry material]. *Jurnal Pendidikan Matematika (Kudus)*, *2*(2), 153–164. https://doi.org/10.21043/jpm.v2i2.6361

Mujib, A., Firmansyah, Siagian, S. S., & M. (2025). The I-CON model in constructing mathematical proof. *Jurnal Eduscience*, *12*(3), 844–858. https://doi.org/10.36987/jes.v12i3.7240

Mujib, A. F. (2022). Improvement habits of minds in constructing mathematical proof using DNR-model. *Aip Conference Proceedings*, *020038*(May 2017). https://doi.org/https://doi.org/10.1063/5.0096095

Mustangin. (2015). Representasi konsep dan peranannya dalam pembelajaran matematika di sekolah [Concept representation and its role in mathematics learning in schools]. *Jurnal Pendidikan Matematika*, *I*(1), 15–21. https://doi.org/10.33474/jpm.v1i1.405

Nauli, N., Harisman, Y., Armiati, & Y. (2024). Junior high school students' habits of mind in solving mathematical problems. *Jurnal Pendidikan Matematika*, *18*(1), 39–58. https://doi.org/10.22342/jpm.v18i1.pp39-58

Nisa, F. B., Mukhlis, M., & Maswar, M. (2020). Analisis hubungan antara kecerdasan logis matematis dengan kemampuan komunikasi matematis siswa [Analysis of the relationship between logical mathematical intelligence and students' mathematical communication abilities]. *Alifmatika: Jurnal Pendidikan Dan Pembelajaran Matematika*, *2*(2), 199–211. https://doi.org/10.35316/alifmatika.2020.v2i2.199-211

OECD. (2022). *PISA 2022 Results (Volume I): The state of learning and equity in education: Vol. I*. https://doi.org/10.1787/53f23881-en

Purwasih, R., Sari, N. R., & Agustina, S. (2018). Analisis kemampuan literasi matematik dan mathematical habits of mind siswa SMP pada materi bangun ruang sisi datar [Analysis of mathematical literacy skills and mathematical habits of mind of junior high school students on the material of flat-sided geometric shapes]. Numeracy, 5(1), 67-76. https://doi.org/10.46244/numeracy.v5i1.318

Rahmah, M., & Zulfadewina, Z. (2025). Analysis of fifth-grade students' numeracy literacy skills in Mathematics. *Curricula: Journal of Curriculum Development, 4*(2), 1847-1858. https://ejournal-education.upi.edu/curricula/article/view/561

Siagian, S. S., Mujib, A., & Zahari, C. L. (2022). Analisis tingkat kecemasan matematika dalam pembentukan konsep image siswa [Analysis of the level of mathematical anxiety in the formation of students' image concepts]. *Paradikma Jurnal Pendidikan Matematika*, *15*(1), 8–13. https://doi.org/10.24114/paradikma.v15i1.34569

Siregar, N. F. (2021). Pemahaman konsep matematika siswa smp melalui pendekatan realistic mathematics education [Understanding of junior high school students' mathematical concepts through a realistic mathematics education approach]. *Jurnal Cendekia, 5*(2), 1919-1927. https://doi.org/10.31004/cendekia.v5i2.635

Suri, I. R. A., Supriadi, N., Yuliana, I., & Suherman, S. (2024). Concept understanding and mathematical representation ability: dmr model with a reciprocal teaching approach. *Alifmatika: Jurnal Pendidikan Dan Pembelajaran Matematika*, *6*(2), 205–217. https://doi.org/10.35316/alifmatika.2024.v6i2.205-217

Tashtoush, M. A. (2022). The effect of a training program based on TIMSS to developing the levels of habits of mind and mathematical reasoning skills among pre-service mathematics teachers. *Eurasia Journal of Mathematics, Science and Technology Education*, *18*(11). https://doi.org/https://doi.org/10.29333/ejmste/12557

Wahyuni, G., Mujib, A., & Zahari, C. L. (2022). Analisis kemampuan berpikir visual siswa ditinjau dari adversity quotient [Analysis of students' visual thinking abilities in terms of adversity quotient]. *JUPE : Jurnal Pendidikan Mandala*, *7*(2), 289–295. https://doi.org/10.58258/jupe.v7i2.3335

Zahari, C. L., & Razali, M. (2022). Penjumlahan dan pengurangan penanaman konsep bilangan bulat dengan pendekatan pembelajaran matematika realistik [Addition and subtraction of the concept of integers with a realistic mathematics learning approach]. *Jurnal Cendekia: Jurnal Pendidikan Matematika*, *6*(2), 2040–2047. https://doi.org/10.31004/cendekia.v6i2.1377