Alifmatika: Jurnal Pendidikan dan Pembelajaran Metematika
Volume 7, Issue 2, 391-408, December 2025
e-ISSN: 2715-6109 | p-ISSN: 2715-6095
https://journal.ibrahimy.ac.id/index.php/Alifmatika

Super (*a, d*)-hyperedge antimagic total labeling on hypergraphs of the volcano graph, semi parachute graph, and comb product graph

Robiatul Adawiyah[1*], Okta Endri Asari[2], Dafik Dafik[3], Arika Indah Kristiana[4], Rafiantika Megahnia Prihandini[5], A Rohini[6]

[1,2,3,4,5]Department of Mathematics Education, University of Jember, East Java 68121, Indonesia
[6]Department of Science and Humanities, KGISL Institute of Technology, Coimbatore, Tamil Nadu 641035, India
[1*]robiatul@unej.ac.id, [2]oktaendri25@gmail.com, [3]d.dafik@unej.ac.id, [4]arika.fkip@unej.ac.id, [5]arika.fkip@unej.ac.id, [6]rohinianbu@gmail.com

Received: July 14, 2025 | Revised: December 6, 2025 | Accepted: December 11, 2025 | Published: December 15, 2025
*Corresponding author

Abstract:
In graph theory, understanding the labeling of graphs and hypergraphs provides valuable insights into their structural properties and applications. A hypergraph generalizes the notion of a conventional graph, defined as a mathematical structure built from a vertex set V and a hyperedge set E, where each hyperedge is allowed to connect two or more vertices simultaneously. The essential distinction between a graph and a hypergraph lies in their edges. While in a graph a single edge connects exactly two vertices, in a hypergraph a single hyperedge may connect any number of vertices, including two. A hypergraph H is considered to admit a super (a, d) -hyperedge antimagic total labeling, such that the vertex label functions f: $V(H) \to 1, 2, 3, \ldots, V(H)$ then f: $E(H) \to V(H) + 1, \ldots, V(H) + V(H)$ and weight $w(e_i) = \sum f(e_i) + \sum f(V_{i,j})$, where i denotes the number of hyperedges, j represents the number of vertices contained in a hyperedge, and e_i refers to the set of vertices and its associated edges with weight $w(e_i)$ for each hyperedge. A super (a, d) -hyperedge antimagic total labeling is formulated as a labeling scheme based on arithmetic progressions, where □ serves as the initial value and d denotes the common difference between consecutive labels. In this scheme, the total weight of a hyperedge is determined by deriving from the sum of the vertex labels and the label of the respective hyperedge. The labels are arranged in an arithmetic sequence, ensuring that each hyperedge has a distinct weight. This study focuses on several special classes of hypergraphs, namely, the volcano graph, the semi-parachute graph, and the comb product of graphs, to implement and examine the characteristics of the super (a, d)-hyperedge antimagic total labeling. By focusing on these graph classes, the study contributes to combinatorics by offering a deeper understanding of hypergraph labeling schemes and their potential applications in network theory, coding theory, and data modeling.

Keywords: Hypergraph; Super (*a, d*)-Hyperedge Antimagic Total Labeling.

How to Cite: Adawiyah, R., Asari, O. E., Dafik, D., Kristiana, A. I., Prihandini, R. M., & Rohini, A. (2025). Super (a, d)-hyperedge antimagic total labeling on hypergraphs of the volcano graph, semi parachute graph, and comb product graph. *Alifmatika: Jurnal Pendidikan dan Pembelajaran Matematika*, 7(2), 391-408. https://doi.org/10.35316/alifmatika.2025.v7i2.391-408

Introduction

Hypergraphs are mathematical structures that generalize the concept of graphs (Dasar, 2020). In traditional graphs, the basic elements are vertices and edges, where each edge connects exactly two vertices. However, in a hypergraph, a single edge, or so-called hyperedge, can connect more than two vertices (Bretto, 2013). The set of vertices in a hypergraph H can be denoted as $V(H) = \{v_1. v_2. .... v_m\}$ and the set of hyper edges can be denoted as $\varepsilon(H) = \{e_1. e_2. .... e_m\}$. In hypergraph theory, there are two important parameters that represent the size and structure of hypergraphs, namely order and size (Dasar et al., 2020). The number of vertices in a hypergraph H is referred to as its order, denoted by $|V(H)|$, whereas the number of hyperedges is referred to as its size, denoted by $|E(\mathrm{H})|$ (Tuczy, 2019). Understanding these two characteristics is an important basis in analyzing the complexity of hypergraph structures, especially in the implementation of labeling theory.

One of the central areas of study in graph and hypergraph theory is graph labeling, which denotes the act of assigning numerical values to vertices, edges, or both, in accordance with specified mathematical rules or structural properties (A. Gallian, 2022). According to the nature of their mapping domains, both graph labeling and hypergraph labeling can be classified into three main categories: vertex labeling, edge labeling, *and* total labeling. (Adawiyah, M. Prihandini, et al., 2023). Vertex labeling maps labels from the set of integers to vertices, edge labeling maps labels from the set of integers to edges, while total labeling covers both elements simultaneously (Bahmanian & Sajna, 2015).

Among the various types of labeling studied, antimagic labeling is among the most interesting because it emphasizes the uniqueness of the resulting weight (You et al., 2018). In this type of labeling, each vertex or edge is assigned a label such that the weight of an element, defined as the sum of the labels of the vertices connected to a given edge, is distinct, ensuring that no two elements share the same weight (Hartsfield & Ringel, 1990).

A further extension of antimagic labeling is the total antimagic (a,d)-edge labeling, in which the weights of the edges are arranged to generate an arithmetic sequence whose initial element is a with successive terms differing d. (Adawiyah, Makhfudloh, et al., 2023) (Adawiyah & M. Prihandini, 2023). The term "super" refers to the additional condition that the smallest label must first be assigned to the vertex before being applied to the edge (Dafik et al., 2009). Research on this type of labeling has continued to grow in the last decade, especially on graph structures such as paths (Saibulla & Pushpam, 2025), cycles (Smita, 2021; Series, 2016), stars (Muthuselvi & Devi, 2025; Arumugam & Nalliah, 2012), fans (Prihandini & Adawiyah, 2022; Dafik et al., 2016), and wheels (Nadzima & Martini, 2019; Sumarno et al., 2015). These studies have significantly contributed to our understanding of antimagic labeling, particularly in the context of ordinary graphs, by providing methods for assigning distinct edge weights from arithmetic sequences.

However, while these studies advance the theory for standard graph structures, there remains a notable gap in the application of total antimagic labeling to hypergraphs. The existing literature has yet to explore how the principles established for ordinary graphs can be extended to hypergraphs, which have distinct structural properties due to their hyperedges. Thus, while research on ordinary graphs has laid a solid foundation, it leaves a critical gap in applying these labeling schemes to more complex structures, such as hypergraphs, which is the focus of this study.

As graph studies evolve, the need to extend the concept of labeling to more complex structures such as hypergraphs becomes increasingly relevant (Adawiyah, M. Prihandini, et al., 2023). Hypergraphs present new challenges in antimagic labeling due to their hyperspace nature, which can involve more than two vertices (Venkatraman et al., 2018). It calculates weights and label distributions much more complex than in regular graphs, so a special approach is needed to ensure the antimagic property in this context.

Some early research has marked the importance of labeling on hypergraphs. Sonntag (2002) was one of the pioneers who explored the labeling of antimagic points on hypergraphs (Sonntag, 2002). Furthermore, Parag and Elgammal (2011) developed a guided hypergraph labeling approach in the context of computer vision and pattern recognition (Parag & Elgammal, 2011). Javaid (2013) then compiled a comprehensive review of various labeling techniques on graphs and hypergraphs (Muhammad, 2013), which opens the door to further research on complex structures such as multilevel hypergraphs and graph products.

Recent studies have begun to lead to the application of total labeling of super (a, d) hyperedges on various hypergraph structures. Dafik et al. (2024) studied this type of labeling specifically on path hypergraphs and triangular ladder hypergraphs. This paper aims to explore the total super (a, d)-hyperedge labeling on three particular hypergraph structures: the volcano graph, the semi parachute graph, and the comb product of graphs. By investigating these structures, we aim to extend the understanding of edge labeling in more complex and diverse graph configurations.

Definition 1. (Dafik et al., 2024) *Let H = (V, E) represents a simple connected hypergraph. The hypergraph H is called super (a, d)-hyperedge antimagic total labeling. A vertex label functions f: V(H) → 1, 2, 3, ..., V(H) then f: E(H) → V(H) + 1, ..., V(H) + V(H) and weight $w(e_i) = \sum f(e_i) + \sum f(V_{i,j})$, where i denotes the number of hyperedges, j represents the number of vertices contained in a hyperedge, and e_i refers to the set of vertices and its associated edge with weight $w(e_i)$ for each hyperedge.*

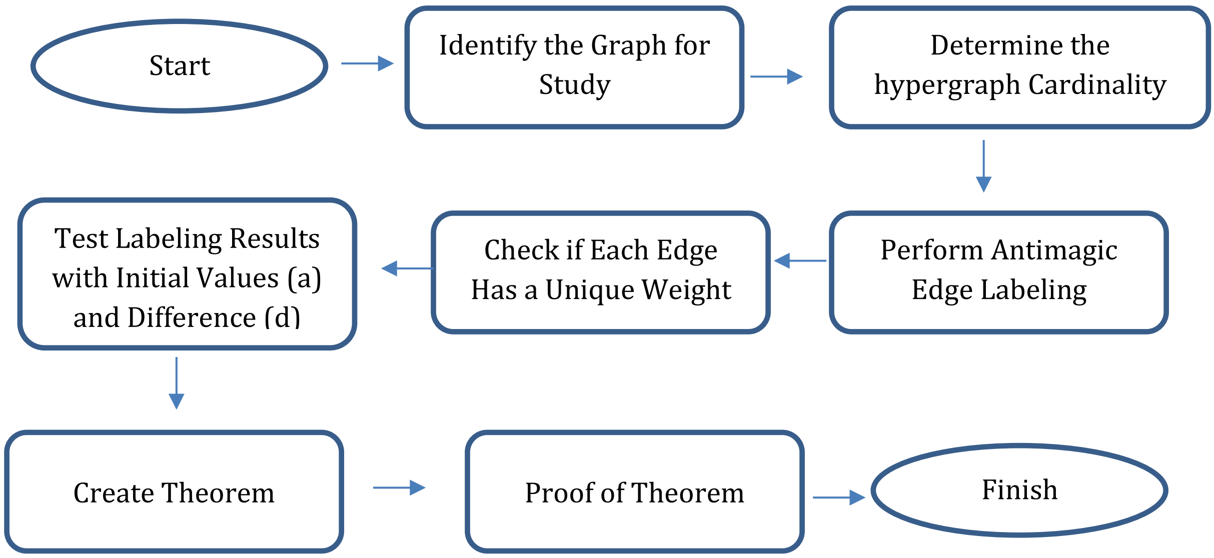
Theorem 1. Dafik et al. (2024), if (p, q) -hypergraph is super (a, d) -hyperedge antimagic total labeling, then:

$$d \le \frac{(pG - pH)pH + (qG - qH)qH}{s - 1}$$

For $pG = |V(H)|, qG = |E(H)|, pH = |V'(H)|, qH = |E'(H)|$, and $s = |H_i|$

Research Methods

The research employs two methodological approaches: pattern recognition and axiomatic deductive. Pattern recognition is utilized to identify and establish regularities in the super (a, d)-hyperedge antimagic total labeling of the hypergraphs under investigation. This axiomatic, deductive approach, grounded in the principles of mathematical logic, is then applied to prove the resulting findings formally. The research procedures are carried out in six stages. First, the cardinalities of both the vertex and hyperedge sets are determined. Second, a superbound for the difference d is established. Third, vertex labels, hyperedge labels, and total labels are assigned. Fourth, the detected labeling pattern is tested against the specified bound of d; if the condition is not satisfied, the process is repeated from the preceding step, whereas if it is satisfied, the procedure proceeds further. Fifth, functions are constructed for vertex labeling, hyperedge labeling, and total weight. Finally, the sixth stage formulates the theorems and provides rigorous proofs.



Picture 1. Research Flowchart

Results and Discussions

In this work, three theorems are formulated and proved within the framework of super (*a*, *d*)-hyperedge antimagic total labeling. The detailed statements and proofs are provided in relation to the structure associated with the volcano hypergraph (*Vn*) ($\mathcal{V}_n$), semi parachute hypergraph ($\mathcal{SP}_n$), and comb hypergraph ($\mathcal{CB}_n$). We prove that the volcano hypergraph ($\mathcal{V}_n$) for $n \geq 2$, the semi parachute hypergraph ($\mathcal{SP}_n$) for $n \geq 3$, and the and comb hypergraph ($\mathcal{CB}_n$) for $n \geq 4$ can be assigned an (*a*, *d*)-hyperedge antimagic total labeling for values of *d* belonging to the set {0, 1, 2}.

Theorem 2. Volcano hypergraph *Vn* with $n \geq 2$ has a super (*a*, *d*)-hyperedge antimagic total labeling where $(a.d) \in \left\{ \left(\frac{m}{4}(2mn - 5m + 5n + 10) + mn - m + 5n + 1, 0 \right). \left(\frac{m+1}{4}(2mn + 8m + 8n - 6) + 3, 1 \right). \left(\frac{m}{4}(2mn + 6m + 6n - 2) + mn - m + 4n + 7, 2 \right) \right\}$

Proof. Let be a volcano hypergraph with vertex set $V(V_n)$ and hyperedge set $E(V_n)$. Its vertices are $V(V_n) = \{x\} \cup \{x_i;\ 1 \leq i \leq 3\} \cup \{y_i;\ 1 \leq i \leq n\} \cup \{x_{i,j};\ 1 \leq i \leq 3, 1 \leq j \leq m\} \cup \{y_{i,j};\ 1 \leq i \leq n, 1 \leq j \leq m\}$ and its hyperedges are $E(V_n) = \{e_{1,i};\ 1 \leq i \leq 3\} \cup \{e_{2,j};\ 1 \leq i \leq n\}$. The cardinalities of vertices and hyperedges in the volcano hypergraph V_n are $|V(V_n)| = (n + 3)(m + 1)$ and $|E(V_n)| = n + 3$.

Case 1, $d = 0$

$V_{8,4}$ for *m* is even, $m \geq 2$ and $n \geq 2$, let a mapping f_1 from $V(H) \rightarrow 1, 2, ..., (n + 3)(m + 1)$ as follows:

$f_1(x) = 2$

$$f_1(x_i)=\begin{cases} i & ;\forall\, i=1 \\ i+1 & ;\forall\, i=2 \end{cases}$$

$$f_1(y_i)=i+3 \quad ;\forall\, 1\le i\le n$$

$$f_1(x_{i,j})=\begin{cases} jn+\dfrac{7j}{2}+i-1\,;\forall\, j\equiv 1\,(mod\ 2),1\le i\le 3 \\ jn+3j+n-i+4\,;\forall\, j\equiv 0\,(mod\ 2),1\le i\le 3 \end{cases}$$

$$f_1(y_{i,j})=\begin{cases} jn+\dfrac{7j}{2}+i+2\,;\forall\, j\equiv 1\,(mod\ 2),1\le i\le n \\ jn+3j+n-i+1\,;\forall\, j\equiv 0\,(mod\ 2),1\le i\le n \end{cases}$$

$$f_1(e_{1,i})=mn-m+5n-i-1\,;\forall\, 1\le i\le 3$$

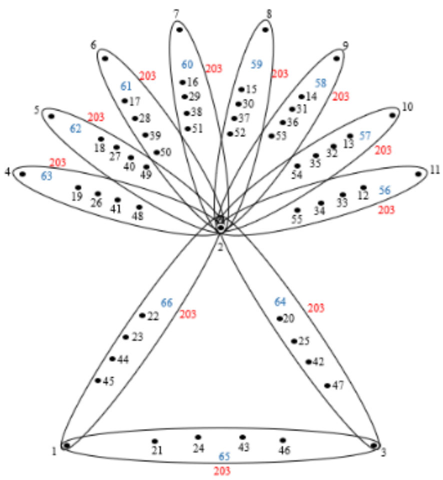
$$f_1(e_{2,i})=mn-m+5n-i-4\,;\forall\, 1\le i\le n$$

Clearly, the labeling f_1 from $V(H)\rightarrow 1, 2, \ldots\ldots, (n+3)(m+1)$ is a bijection. Under the specified labeling, each edge's weight is represented by f_1 sets:

$$W_{f_1}^{1}(e_{1,i})=\frac{m}{4}(2mn-5m+5n+10)+mn-m+5n+1$$

$$W_{f_1}^{2}(e_{2,i})=\frac{m}{4}(2mn-5m+5n+10)+mn-m+5n+1$$

Total weight of $\bigcup_{r=1}^{2} W_{f_1}^{r}=\left\{\frac{m}{4}(2mn-5m+5n+10)+mn-m+5n+1,\ldots.\frac{\boldsymbol{m}}{\boldsymbol{4}}(\boldsymbol{2mn-5m+5n+10})+mn-m+5n+1\right\}$ have the same elements, then hypergraph V_n proven $\left(\frac{m}{4}(2mn-5m+5n+10)+mn-m+5n+1\,,0\right)$-hyperedge antimagic total labeling.



Picture 2. Super (203,0)-Hyperedge Antimagic Total Labeling on $V_{4,8}$

Picture 2 illustrates the application of the Super-Hyperedge Antimagic Total Labeling on $\boldsymbol{V_{4,8}}$, where the initial label $\boldsymbol{a}$ is 203, and the common difference $\boldsymbol{d}$ is 0. This labeling scheme assigns distinct weights to the hyperedges of the hypergraph so that each hyperedge has a unique total weight. The specific choice of ***a* = 203** and ***d* = 0** ensures that the labeling follows a constant value.

Case 2. $d = 1$

$V_{8,5}$ for m is odd, $m \geq 1$ and $n \geq 2$, let a mapping f_2 from $V(H) \to 1, 2, \ldots, (n+3)(m+1)$ as follows:

$$f_2(x) = 2$$

$$f_2(x_i) = \begin{cases} i\,; \forall\, i = 1 \\ i+1\,; \forall\, i = 2 \end{cases}$$

$$f_2(y_i) = i + 3 \;\; ; \forall\, 1 \leq i \leq 2$$

$$f_2(x_{i,j}) = \begin{cases} jn + \frac{7j}{2} + i - 1\,; \forall\, j \equiv 1\,(mod\ 2), 1 \leq i \leq 3 \\ jn + 3j + n - i + 4\,; \forall\, j \equiv 0\,(mod\ 2), 1 \leq i \leq 3 \end{cases}$$

$$f_2(y_{i,j}) = \begin{cases} jn + \frac{7j}{2} + i + 2\,; \forall\, j \equiv 1\,(mod\ 2), 1 \leq i \leq n \\ jn + 3j + n - i + 1\,; \forall\, j \equiv 0\,(mod\ 2), 1 \leq i \leq n \end{cases}$$

$$f_2(e_{1,i}) = mn - m + 4n + i - 1\,; \forall\, 1 \leq i \leq 3$$

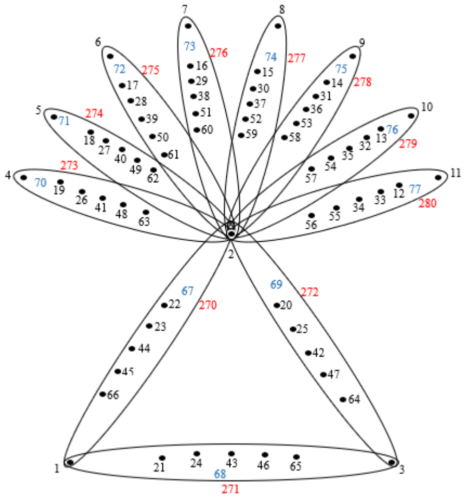
$$f_2(e_{2,i}) = mn - m + 4n + i + 2\,; \forall\, 1 \leq i \leq n$$

Clearly, the labeling f_2 from $V(H) \to 1, 2, \ldots, (n+3)(m+1)$ is a bijection. Under the labeling f_2, each edge weight corresponds to the set:

$$W_{f_2}^{1}(e_{1,i}) = \frac{m+1}{4}(2mn + 8m + 8n - 6) + i + 2$$

$$W_{f_2}^{2}(e_{2,i}) = \frac{m+1}{4}(2mn + 8m + 8n - 6) + i + 5$$

Total weight $\bigcup_{r=1}^{2} W_{f_2}^{r} = \left\{ \frac{m+1}{4}(2mn + 8m + 8n - 6) + 3, \frac{m+1}{4}(2mn + 8m + 8n - 6) + 4, \frac{m+1}{4}(2mn + 8m + 8n - 6) + 5, \ldots \right\}$ has consecutive elements, then hypergraph V_n is proven $\left(\frac{m+1}{4}(2mn + 8m + 8n - 6) + 3, 1 \right)$-hyperedge antimagic total labeling.



Picture 3. Super (270,0)-Hyperedge Antimagic Total Labeling on $V_{5,8}$

Picture 3 is an illustration of Super-Hyperedge Antimagic Total Labeling, which has an a value of a is 207 and d value of 0.

Case 3. d = 2

$V_{6,8}$ for m is even, $m \geq 2$ and $n \geq 2$, let a mapping f_3 from $V(H) \rightarrow 1, 2, \ldots, (n+3)(m+1)$ as follows:

$$f_3(x) = 2$$

$$f_3(x_i) = \begin{cases} i \,;\forall\, i \equiv 1 \,(mod\ 2), 1 \leq i \leq 1 \\ i+1 \,;\forall\, i \equiv 0 \,(mod\ 2), 1 \leq i \leq 2 \end{cases}$$

$$f_3(y_i) = i+3\,;\ \forall\ 1 \leq i \leq 2$$

$$f_3(x_{i,j}) = \begin{cases} \dfrac{jn+n}{2} + i + 3 \,;\forall\ j \equiv 1 \,(mod\ 2), 1 \leq i \leq 3 \\ 2jn + \dfrac{j}{2} - i - 1 \,;\forall\ j \equiv 0 \,(mod\ 2), 1 \leq i \leq 3 \end{cases}$$

$$f_3(y_{i,j}) = \begin{cases} \dfrac{jn+n}{2} + i + 6 \,;\forall\ j \equiv 1 \,(mod\ 2), 1 \leq i \leq n \\ 2jn + \dfrac{j}{2} - i - 2 \,;\forall j \equiv 0 \,(mod\ 2), 1 \leq i \leq n \end{cases}$$

$$f_3(e_{1,i}) = mn - m + 4n + i + 3\,;\forall\, 1 \leq i \leq 3$$

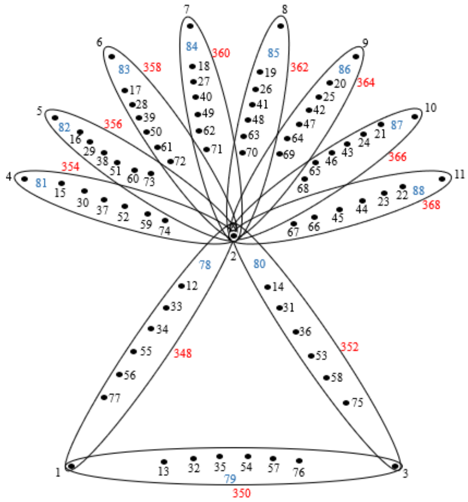
$$f_3(e_{2,i}) = mn - m + 4n + i + 5\,;\forall\, 1 \leq i \leq n$$

Clearly, the labeling f_3 from $V(H) \rightarrow 1, 2, \ldots, (n+3)(m+1)$ is a bijection. Under the labeling f_3, each edge weight corresponds to the sets:

$$W^1_{f_3}(e_{1,i}) = \frac{m}{4}(2mn + 6m + 6n - 2) + mn - m + 4n + 2i + 5$$

$$W^2_{f_2}(e_{2,i}) = \frac{m}{4}(2mn + 6m + 6n - 2) + mn - m + 4n + 2i + 10$$

Total weight $\bigcup_{r=1}^{2} W_{f_3}^{r} = \left\{ \frac{m}{4}(2mn+6m+6n-2)+mn-m+4n+7, \frac{m}{4}(2mn+6m+6n-2)+ mn-m+4n+9. \frac{m}{4}(2mn+6m+6n-2)+mn-m+4n+11, \ldots \right\}$ has consecutive elements, then hypergraph V_n is proven $\left(\frac{m}{4}(2mn+6m+6n-2)+mn-m+4n+7, 2 \right)$-hyperedge antimagic total labeling.



Picture 4. Super (348,0)-Hyperedge Antimagic Total Labeling on $V_{6,8}$

Picture 4 illustrates the Super (348,0)-Hyperedge Antimagic Total Labeling applied to the hypergraph $V_{6,8}$, with the initial label a = 348 and the common difference d = 0.

Theorem 3. Semi-parachute hypergraph $\mathcal{SP}_n$ with $n \geq 3$ admits a super $(a.d)$-hyperedge antimagic total labeling where $(a,d) \in \left\{ \left(\frac{m}{4}(5mn-2m+12n+2)+3mn-m+5n+2, 0 \right). \left(\frac{m+1}{4}(5mn-m+16n+7)+3, 1 \right). \left(\frac{m}{2}(mn+4m+5n+15)+3mn-m+2n+4, 2 \right) \right\}$

Proof. Let be a semi-parachute hypergraph having vertex set $\boldsymbol{V}(\boldsymbol{\mathcal{SP}_n})$ hyperedge set $\boldsymbol{\mathcal{E}}(\boldsymbol{\mathcal{SP}_n})$. The vertex set is defined as $(\boldsymbol{\mathcal{SP}_n}) = \{x\} \cup \{x_i \,;\, 1 \leq i \leq n\} \cup \{U_i; 1 \leq i \leq n-1\} \cup \left\{ x_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m \right\} \cup \{ y_{i,j}; 1 \leq i \leq 2n-1, 1 \leq j \leq m \}$ and the hyperedge set is expressed as $\boldsymbol{\mathcal{E}}(\boldsymbol{\mathcal{SP}_n}) = \{\boldsymbol{e_{1.i}};\, \mathbf{1} \leq \boldsymbol{i} \leq \boldsymbol{n}\} \cup \{\boldsymbol{e_{2.i}};\, \{\mathbf{1} \leq \boldsymbol{i} \leq \mathbf{2}\boldsymbol{n} - \mathbf{1}\}$. The cardinalities of vertices and hyperedges in the semi parachute hypergraph $\boldsymbol{\mathcal{SP}_n}$ are $|\boldsymbol{V}(\boldsymbol{\mathcal{SP}_n})| = 3mn-m+n+3$ and $|\boldsymbol{\mathcal{E}}(\boldsymbol{\mathcal{SP}_n})| = 3n-1$.

Case 1. d = 0

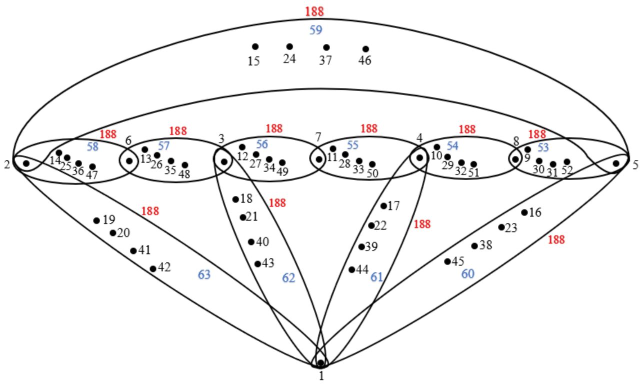
$\mathcal{SP}_{4.4}$ for m is even, $m \geq 2$ and $n \geq 3$, let a mapping f_4 from $V(H) \rightarrow \{1, 2, \ldots, 3mn-m+n+3\}$ as follows:

$$f_4(x) = 1$$
$$f_4(x_i) = i + 1 ; \forall 1 \le i \le n$$
$$f_4(u_i) = \frac{n}{2} + i + 3 ; ?1 \le i \le n - 1$$
$$f_4(x_{i,j}) = \begin{cases} \frac{5jn + j + 5n + 1}{2} - i - 1 ; \forall\ j \equiv 1 \,(mod\ 2), 1 \le i \le n \\ \frac{5jn + j}{2} + i - 2 ; \forall\ j \equiv 0 \,(mod\ 2), 1 \le i \le n \end{cases}$$
$$f_4(y_{i,j}) = \begin{cases} \frac{5jn + j + 5n + 1}{2} - i - 5 ; \forall\ j \equiv 1 \,(mod\ 2), 1 \le i \le 2n - 1 \\ \frac{5jn + j}{2} + i + 2 ; \forall\ j \equiv 0 \,(mod\ 2), 1 \le i \le 2n - 1 \end{cases}$$
$$f_4(e_{1,i}) = 3mn - m + 5n - i ; \forall 1 \le i \le n$$
$$f_4(e_{2,i}) = 3mn - m + 4n - i ; \forall 1 \le i \le 2n - 1$$

Clearly, the labeling f_4 from $V(H) \to \{1, 2, ..., 3mn - m + n + 3\}$ is a bijection. Under the sprecified labeling, each edge's weight is represented by f_4 sets:

$$W^1_{f_4}(e_{1,i}) = \frac{m}{4}(5mn - 2m + 12n + 2) + 3mn - m + 5n + 2$$
$$W^2_{f_4}(e_{2,i}) = \frac{m}{4}(5mn - 2m + 12n + 2) + 3mn - m + 5n + 2$$

Total weight of $\bigcup_{r=1}^{2} W^r_{f_4} = \left\{ \frac{m}{4}(5mn - 2m + 12n + 2) + 3mn - m + 5n + 2, \ldots, \boldsymbol{\frac{m}{4}(5mn - 2m + 12n + 2) + 3mn - m + 5n + 2} \right\}$ have the same elements, then hypergraph $\boldsymbol{\mathcal{SP}_n}$ proven $\left(\frac{m}{4}(5mn - 2m + 12n + 2) + 3mn - m + 5n + 2, 0 \right)$-hyperedge antimagic total labeling.



Picture 5. Super (188,0)-Hyperedge Antimagic Total Labeling on $\mathcal{SP}_{4.4}$

Picture 5 illustrates the Super (188,0)-Hyperedge Antimagic Total Labeling applied to the hypergraph $\boldsymbol{\mathcal{SP}_{4.4}}$, where the initial label a = 188 and the common difference d = 0.

Case 2. $d = 1$

$\mathcal{SP}_{5.4}$ for m is even, $m \geq 1$ and $n \geq 3$, let a mapping f_4 from $V(H) \rightarrow \{1, 2, ..., 3mn - m + n + 3\}$ as follows:

$f_5(x) = 1$

$f_5(x_i) = i + 1 ; \forall\, 1 \leq i \leq n$

$f_5(u_i) = \frac{n}{2} + i + 3 ; \forall\, 1 \leq i \leq n - 1$

$$f_5(x_{i,j}) = \begin{cases} \frac{5jn + j + 5n + 1}{2} - i - 1 ; \forall\, j \equiv 1 \,(mod\ 2), 1 \leq i \leq n \\ \frac{5jn + j}{2} + i - 2 ; \forall\, j \equiv 0 \,(mod\ 2), 1 \leq i \leq n \end{cases}$$

$$f_5(y_{i,j}) = \begin{cases} \frac{5jn + j + 5n + 1}{2} - i - 5 ; \forall\, j \equiv 1 \,(mod\ 2), 1 \leq i \leq 2n - 1 \\ \frac{5jn + j}{2} + i + 2 ; \forall\, j \equiv 0 \,(mod\ 2), 1 \leq i \leq 2n - 1 \end{cases}$$

$f_5(e_{1,i}) = 3mn - m + 2n + i ; \forall\, 1 \leq i \leq n$

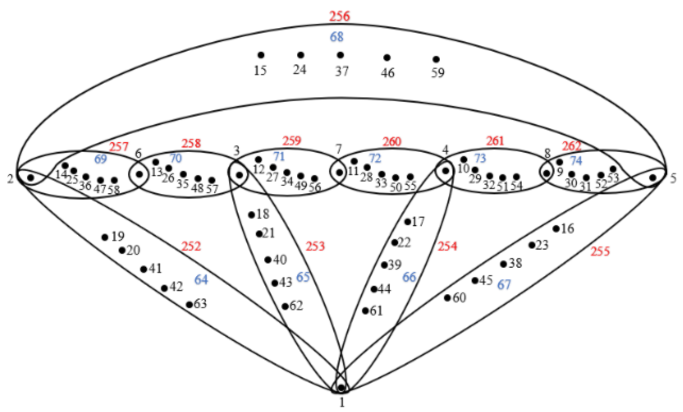
$f_5(e_{2,i}) = 3mn - m + 3n + i ; \forall\, 1 \leq i \leq 2n - 1$

Clearly, the labeling f_5 from $V(H) \rightarrow \{1, 2, \ldots, 3mn - m + n + 3\}$ is a bijection. Under the labeling f_5, each edge weight corresponds to the sets:

$$W_{f_5}^{1}(e_{1,i}) = \frac{m+1}{4}(5mn - m + 16n + 7) + i + 2$$

$$W_{f_5}^{2}(e_{2,i}) = \frac{m+1}{4}(5mn - m + 16n + 7) + i + 6$$

Total weight $\bigcup_{r=1}^{2} W_{f_5}^{r} = \left\{ \frac{m+1}{4}(5mn - m + 16n + 7) + 3, \frac{m+1}{4}(5mn - m + 16n + 7) + 4, \frac{m+1}{4}(5mn - m + 16n + 7) + 5, \ldots \right\}$ has consecutive elements, then hypergraph $\mathcal{SP}_n$ is proven $\left(\frac{m+1}{4}(5mn - m + 16n + 7) + 3, 1 \right)$-hyperedge antimagic total labeling.



Picture 6. Super (252,1)-Hyperedge Antimagic Total Labeling on $\mathcal{SP}_{5.4}$

Picture 6 illustrates the Super (252,1)-Hyperedge Antimagic Total Labeling applied to the hypergraph $\mathcal{SP}_{5.4}$, where the initial label a = 252 and the common difference d = 1.

Case 3. d = 2
$\mathcal{SP}_{6.4}$ for m is even, $m \geq 2$ and $n \geq 3$, let a mapping f_6 from $V(H) \rightarrow \{1, 2, ..., 3mn - m + n + 3\}$ as follows:

$$f_6(x) = 1$$

$$f_6(x_i) = i + 1, \forall\, 1 \leq i \leq n$$

$$f_6(u_i) = \frac{n}{2} + i + 3, \forall\, 1 \leq i \leq n - 1$$

$$f_6(x_{i,j}) = \begin{cases} 4jn - \frac{3j-3}{2} + 2n + i + 3\,; \forall\, j \equiv 1 (mod\ 2), 1 \leq i \leq n \\ 4jn - \frac{3j}{2} - i + 2\,; \forall\, j \equiv 1 (mod\ 2), 1 \leq i \leq n \end{cases}$$

$$f_6(y_{i,j}) = \begin{cases} 4jn - \frac{3j-3}{2} + 2n + i + 7\,; \forall\ \ j \equiv 1 (mod\ 2), 1 \leq i \leq 2n - 1 \\ 4jn - \frac{3j}{2} - i - 2\,; \forall\, j \equiv 1 (mod\ 2), 1 \leq i \leq 2n - 1 \end{cases}$$

$$f_6(e_{1,i}) = 3mn - m + 2n + i\,; \forall\, 1 \leq i \leq n$$

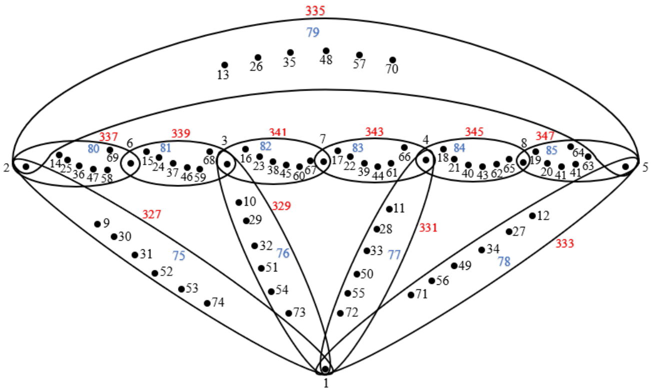
$$f_6(e_{2,i}) = 3mn - m + 3n + i\,; \forall\, 1 \leq i \leq 2n - 1$$

Clearly, the labeling f_6 from $V(H) \rightarrow \{1, 2, ..., 3mn - m + n + 3\}$ is a bijection. Under the specified labeling, each edge's weight is represented by f_6 sets:

$$W_{f_6}^{1}(e_{1,i}) = \frac{m}{2}(mn + 4m + 5n + 15) + 3mn - m + 2n + 2i + 2$$

$$W_{f_6}^{2}(e_{2,i}) = \frac{m}{2}(mn + 4m + 5n + 15) + 3mn - m + 3n + 2i + 6$$

Total weight $\bigcup_{r=1}^{2} W_{f_6}^{r} = \Big\{ \frac{m}{2}(mn + 4m + 5n + 15) + 3mn - m + 2n + 4,\ \frac{m}{2}(mn + 4m + 5n + 15) + 3mn - m + 2n + 6. \boldsymbol{\frac{m}{2}}(mn + 4m + 5n + 15) + 3mn - m + 2n + 8 \ldots \Big\}$ having sequential elements, then hypergraph $\mathcal{SP}_n$ is proven $\left(\frac{m}{2}(mn + 4m + 5n + 15) + 3mn - m + 2n + 4, 2\right)$-hyperedge antimagic total labeling.



Picture 7. Super (327,2)-Hyperedge Antimagic Total Labeling on $\mathcal{SP}_{6.4}$

Picture 7 illustrates the Super (327,2)-Hyperedge Antimagic Total Labeling applied to the hypergraph $\mathcal{SP}_{6.4}$, with the initial label a = 327 and the common difference d = 2.

Theorem 4. Comb hypergraph $\mathcal{CB}_n$ with $n \geq 4$ can be assigned a super (a, d) -hyperedge antimagic total labeling where $(a,d) \in \left\{ \left(\frac{m}{4}(6mn - 14m + 6n - 6) + 2mn - m + 3n + 3, 0 \right). \left(\frac{m+1}{4}(6mn - 13m + 14n - 37) + 3, 1 \right). \left(\frac{m}{2}(mn + 4m + 3n + 3) + 2mn - m + n + 5, 2 \right) \right\}$

Proof. Let be a comb hypergraph having vertex set $V(\mathcal{CB}_n)$ and hyperedge set $\mathcal{E}(\mathcal{CB}_n)$. The vertex set is defined as $(\mathcal{CB}_n) = \left\{ x_i; 1 \leq i \leq \frac{n}{2} \right\} \cup \left\{ y_i; 1 \leq i \leq \frac{n}{2} + 1 \right\} \cup \left\{ x_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m \right\} \cup \left\{ y_{i,j}; 1 \leq i \leq \frac{n}{2}, 1 \leq j \leq m \right\} \cup \left\{ z_{i,j}; 1 \leq i \leq \frac{n}{2} - 1, 1 \leq i \leq m \right\}$ and $\mathcal{E}(\mathcal{CB}_n) = \left\{ e_{1,i}; 1 \leq i \leq n \right\} \cup \left\{ e_{2,i}; 1 \leq i \leq \frac{n}{2}, 1 \leq i \leq m \right\} \cup \left\{ e_{3,i}; 1 \leq i \leq \frac{n}{2} - 1, 1 \leq i \leq m \right\}$, The cardinalities of vertices and hyperedges in the comb hypergraph $\mathcal{CB}_n$ are $|V(\mathcal{CB}_n)| = 2mn - m + n + 1$ and $|\mathcal{E}(\mathcal{CB}_n)| = 3n - 2$,

Case 1. $d = 0$

$\mathcal{CB}_{4.6}$ for m is even, $m \geq 2$ and $n \geq 4$, let a mapping f_7 from $V(H) \rightarrow \{1, 2, ..., 2mn - m + n + 1\}$ as follows:

$$f_7(x_i) = i + 1; \forall\, i \equiv 1 \,(mod\ 2), 1 \leq i \leq \frac{n}{2}$$

$$f_7(y_i) = i - 1; \forall\, i \equiv 0 \,(mod\ 2), 1 \leq i \leq \frac{n}{2} + 1$$

$$f_7(x_{i,j}) = \begin{cases} \frac{3jn + 3j + 3n + 3}{2} - 2i - 1; \forall\ j \equiv 1 \,(mod\ 2), 1 \leq i \leq n \\ \frac{3jn + 3j}{2} + 2i - 4; \forall\ j \equiv 0 \,(mod\ 2), 1 \leq i \leq n \end{cases}$$

$$f_7\left(y_{i,j}\right)=\begin{cases}\frac{3jn+3j+3n+3}{2}-4i\,;\forall\ j\equiv 1\left(mod\ 2\right),1\le i\le \frac{n}{2}\\ \frac{3jn+3j}{2}+4i-5\,;\forall\ j\equiv 0\left(mod\ 2\right),1\le i\le \frac{n}{2}\end{cases}$$

$$f_7\left(z_{i,j}\right)=\begin{cases}\frac{3jn+3j+3n+3}{2}-4i-2\,;\forall\ j\equiv 1\left(mod\ 2\right),1\le i\le \frac{n}{2}-1\\ \frac{3jn+3j}{2}+4i-3\,;\forall j\equiv 0\ (mod\ 2).\ 1\le i\le \frac{n}{2}-1\end{cases}$$

$$f_7\left(e_{1,i}\right)=2mn-m+n+2i\,;\forall\, 1\le i\le n$$

$$f_7\left(e_{2,i}\right)=2mn-m+n+4i-1\,;\forall\, 1\le i\le \frac{n}{2}$$

$$f_7\left(e_{3,i}\right)=2mn-m+n+4i\ +1\ ;\ \forall\, 1\le i\le \frac{n}{2}-1$$

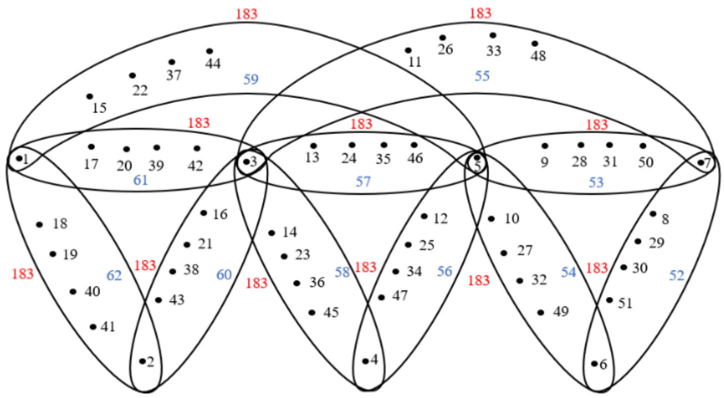
Clearly, the labeling f_7 from $V(H) \to \{1, 2, ..., 2mn - m + n + 1\}$ is a bijection. Under the specified labeling, each edge's weight is represented by f_7 sets:

$$W^1_{f_7}(e_{1,i})=\frac{m}{4}\left(6mn-14m+6n-6\right)+2mn-m+3n+3$$

$$W^2_{f_7}(e_{2,i})=\frac{m}{4}\left(6mn-14m+6n-6\right)+2mn-m+3n+3$$

$$W^3_{f_7}(e_{3,i})=\frac{m}{4}\left(6mn-14m+6n-6\right)+2mn-m+3n+3$$

Total weight of $\bigcup_{r=1}^{3}W^r_{f_7}=\left\{\frac{m}{4}\left(6mn-14m+6n-6\right)+2mn-m+3n+3,\ldots,\ \boldsymbol{\frac{m}{4}(6mn-14m+6n-6)+2mn-m+3n+3}\right\}$ have the same elements, then hypergraph $\boldsymbol{\mathcal{CB}_n}$ proven $\left(\frac{m}{4}\left(6mn-14m+6n-6\right)+2mn-m+3n+3,\,0\right)$-hyperedge antimagic total labeling.



Picture 8. Super (183,0)-Hyperedge Antimagic Total Labeling on $\mathcal{CB}_{4.6}$

Picture 8 illustrates the Super (183,0)-Hyperedge Antimagic Total Labeling applied to the hypergraph $\boldsymbol{\mathcal{CB}_{4.6}}$, where the initial label a = 183 and the common difference d = 0.

Case 2. $d = 1$

$CB_{5.6}$ for m is odd, $m \geq 1$ and $n \geq 4$, let a mapping f_8 from $V(H) \to \{1, 2, ..., 2mn - m + n + 1\}$ as follows:

$$f_8(x_i) = i + 1, \forall\, i \equiv 1\,(mod\ 2), 1 \leq i \leq \frac{n}{2}$$

$$f_8(y_i) = i - 1, \forall\, i \equiv 0\,(mod\ 2), 1 \leq i \leq \frac{n}{2} + 1$$

$$f_8(x_{i,j}) = \begin{cases} \dfrac{3jn + 3j + 3n + 3}{2} - 2i - 1\,; \forall\ j \equiv 1\,(mod\ 2), 1 \leq i \leq n \\ \dfrac{3jn + 3j}{2} + 2i - 4\,; \forall\ j \equiv 0\,(mod\ 2), 1 \leq i \leq n \end{cases}$$

$$f_8(y_{i,j}) = \begin{cases} \dfrac{3jn + 3j + 3n + 3}{2} - 4i\,; \forall\ j \equiv 1\,(mod\ 2), 1 \leq i \leq \dfrac{n}{2} \\ \dfrac{3jn + 3j}{2} + 4i - 5\,; \forall\ j \equiv 0\,(mod\ 2), 1 \leq i \leq \dfrac{n}{2} \end{cases}$$

$$f_8(z_{i,j}) = \begin{cases} \dfrac{3jn + 3j + 3n + 3}{2} - 4i - 2\,; \forall\ j \equiv 1\,(mod\ 2), 1 \leq i \leq \dfrac{n}{2} - 1 \\ \dfrac{3jn + 3j}{2} + 4i - 3\,; \forall\ \boldsymbol{j \equiv 0\ (mod\ 2). 1 \leq i \leq \frac{n}{2} - 1} \end{cases}$$

$$f_8(e_{1,i}) = 2mn - m + n + 2i\,; \forall\, 1 \leq i \leq n$$

$$f_8(e_{2,i}) = 2mn - m + n + 4i - 1\,; \forall 1 \leq i \leq \frac{n}{2}$$

$$f_8(e_{3,i}) = 2mn - m + n + 4i + 1\ ;\ \forall\ 1 \leq i \leq \frac{n}{2}$$

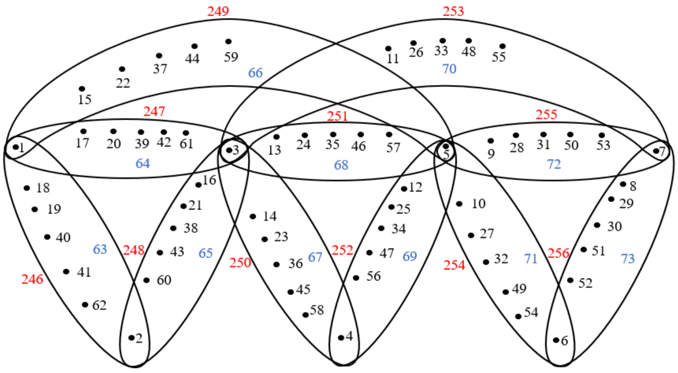
Clearly, the labeling f_8 from $V(H) \to \{1, 2, ..., 2mn - m + n + 1\}$ is a bijection. Under the labeling f_8, each edge weight corresponds to the sets:

$$W^1_{f_8}(e_{1,i}) = \frac{m+1}{4}(6mn - 13m + 14n - 37) + 2i + 1$$

$$W^2_{f_8}(e_{2,i}) = \frac{m+1}{4}(6mn - 13m + 14n - 37) + 4i$$

$$W^3_{f_8}(e_{3,i}) = \frac{m+1}{4}(6mn - 13m + 14n - 37) + 4i + 2$$

Total weight $\bigcup_{r=1}^{3} W^r_{f_8} = \left\{ \frac{m+1}{4}(6mn - 13m + 14n - 37) + 3,\ \frac{m+1}{4}(6mn - 13m + 14n - 37) + 4,\ \boldsymbol{\frac{m+1}{4}}(6mn - 13m + 14n - 37) + 5, \ldots \right\}$ has consecutive elements, then hypergraph $\mathcal{CB}_n$ is proven $\left(\frac{m+1}{4}(6mn - 13m + 14n - 37) + 3, 1 \right)$-hyperedge antimagic total labeling.



Picture 9. Super (246,1)-Hyperedge Antimagic Total Labeling on $\mathcal{CB}_{5.6}$

Picture 9 illustrates the Super (246,1)-Hyperedge Antimagic Total Labeling applied to the hypergraph $\mathcal{CB}_{5.6}$, where the initial label a = 246 and the common difference d = 1.

Case 3. $d = 2$

$\mathcal{CB}_{6.6}$ for m is even, $m \geq 2$ and $n \geq 2$, let a mapping f_9 from $V(H) \rightarrow \{1, 2, ..., 2mn - m + n + 1\}$ as follows:

$$f_9(x_i) = i + 1\,;\, for\ i \equiv 1\,(mod\ 2), 1 \leq i \leq \frac{n}{2}$$

$$f_9(y_i) = i - 1\,;\, for\ i \equiv 0\,(mod\ 2), 1 \leq i \leq \frac{n}{2} + 1$$

$$f_9(x_{i,j}) = \begin{cases} jn - 2j + 2i - 4\,;\forall\ j \equiv 1\,(mod\ 2), 1 \leq i \leq n \\ \dfrac{5jn - j}{2} - 2i + 2\,;\forall\ j \equiv 0\,(mod\ 2), 1 \leq i \leq n \end{cases}$$

$$f_9(y_{i,j}) = \begin{cases} jn - 2j + 4i - 5\,;\forall\ j \equiv 1\,(mod\ 2), 1 \leq i \leq \dfrac{n}{2} \\ \dfrac{5jn - j}{2} - 4i + 3\,;\forall\ j \equiv 0\,(mod\ 2), 1 \leq i \leq \dfrac{n}{2} \end{cases}$$

$$f_9(z_{i,j}) = \begin{cases} jn - 2j + 4i - 3\,;\forall\ j \equiv 1\,(mod\ 2), 1 \leq i \leq \dfrac{n}{2} - 1 \\ \dfrac{5jn - j}{2} - 4i + 1\,;\forall\ j \equiv 0\ (mod\ 2).\ 1 \leq i \leq \dfrac{n}{2} - 1 \end{cases}$$

$$f_9(e_{1,i}) = 2mn - m + n + 2i\,;\forall 1 \leq i \leq n$$

$$f_9(e_{2,i}) = 2mn - m + n + 4i - 1\,;\forall\ 1 \leq i \leq \frac{n}{2}$$

$$f_9(e_{3,i}) = 2mn - m + n + 4i + 1\,;\, for\ 1 \leq i \leq \frac{n}{2} - 1$$

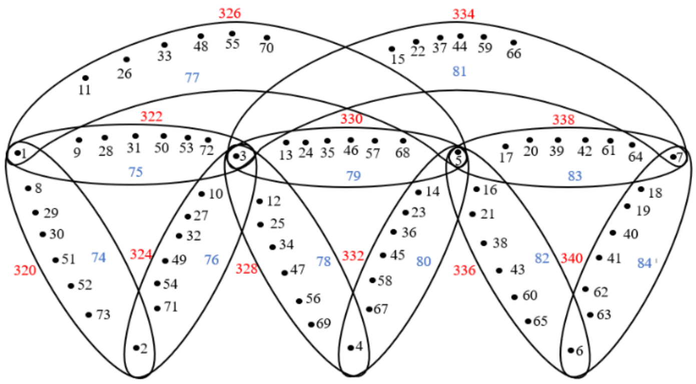
Clearly, the labeling f_9 from $V(H) \rightarrow \{1, 2, ..., 2mn - m + n + 1\}$ is a bijection. Under the specified labeling, each edge's weight is represented by f_9 sets

$$W_{f_9}^1(e_{1,i}) = \frac{m}{2}(mn+4m+3n+3)+2mn-m+n+4i+1$$

$$W_{f_9}^2(e_{2,i}) = \frac{m}{2}(mn+4m+3n+3)+2mn-m+n+8i-1$$

$$W_{f_9}^3(e_{3,i}) = \frac{m}{2}(mn+4m+3n+3)+2mn-m+n+8i+4$$

Total weight $\bigcup_{r=1}^{3} W_{f_9}^r = \left\{ \frac{m}{2}(mn+4m+3n+3)+2mn-m+n+5,\ \frac{\boldsymbol{m}}{\mathbf{2}}(\boldsymbol{mn}+\mathbf{4}\boldsymbol{m}+\mathbf{3}\boldsymbol{n}+\mathbf{3}) +2mn-m+n+7\ \frac{m}{2}(mn+4m+3n+3)+2mn-m+n+9, \ldots \right\}$, Having sequential elements, then hypergraph $\boldsymbol{\mathcal{CB}_n}$ is proven $\left(\frac{m}{2}(mn+4m+3n+3)+2mn-m+n+5, 2\right)$-hyperedge antimagic total labeling.



Picture 10. Super (320,2)-Hyperedge Antimagic Total Labeling on $\mathcal{CB}_{6.6}$

Picture 10 illustrates the Super (320, 2)-Hyperedge Antimagic Total Labeling on the hypergraph $\mathcal{CB}_{6.6}$ with the initial label a = 320 and common difference d = 2.

Conclusions and Suggestions

In this paper, we have proved three theorems related to the antimagic properties of the labeling (a, d)- hyperedge on a hypergraph, with d = 0. 1. 2. This study focuses on three types of hypergraphs, namely the volcano hypergraph, semi parachute hypergraph, and comb hypergraph. These results address the problem of understanding how the choice of label parameters a and d affects the uniqueness of the total weight assignments in different hypergraph structures. As a direction for future research, we suggest that other researchers research super (a, d)-hyperedge antimagic total labeling on other hypergraphs.

Acknowledgements

We want to thank LP2M, University of Jember, for its assistance and support, which provided encouragement and funding during the implementation of this research.

References

A. Gallian, J. (2022). A dynamic survey of graph labeling. *Mathematics & Statistics*, *6*(25), 4-623. https://experts.umn.edu/en/publications/a-dynamic-survey-of-graph-labeling-5

Adawiyah, R., & M. Prihandini, R. (2023). *On local (a, d)-antimagic coloring of some specific classes of graphs title*. https://books.google.co.id/books?hl=id&lr=&id=Vaa7EAAAQBAJ&oi=fnd&pg=PA156&dq=robiatul+adawiyah++local+2023&ots=DYO1i5xJzT&sig=GS68ngjg7ZztQXXHrp-059gqZHE&redir_esc=y#v=onepage&q&f=false

Adawiyah, R., M. Prihandini, R., & H. Agustin, I. (2023). On Local (a, d)-Antimagic Coloring of Some Specific Classes of Graphs. *In Proceedings of the 6th International Conference on Combinatorics, Graph Theory, and Network Topology (ICCGANT 2022)*, *6*(1),156-169. https://doi.org/10.2991/978-94-6463-138-8_14

Adawiyah, R., Makhfudloh, I. I., & M. Prihandini, R. (2023). Local edge (a, d)–antimagic coloring on sunflower, umbrella graph and its application. *Alifmatika: Jurnal Pendidikan dan Pembelajaran Matematika,* *5*(1), 70-81. https://doi.org/10.35316/alifmatika.2023.v5i1.70-81

Arumugam, S., & Nalliah, M. (2012). *of friendship graphs*. *53*, 237–243.

Bahmanian, M. A., & Sajna, M. (2015). *Hypergraphs : connection and separation*. 1–31.

Bretto, A. (2013). *Hypergraph Theory*. https://link.springer.com/book/10.1007/978-3-319-00080-0

Dafik, H. Agustin, I., & Faridatun, K. (2016). Super (a, d)− F n-antimagic total labeling for a connected and disconnected amalgamation of fan graphs. *AIP Conference Proceedings, 1707*(1), 1-12. https://doi.org/10.1063/1.4940804

Dafik, Jannah, E. S. W., Agustin, I. H., Venkatraman, S., Mursyidah, I. L., Alfarisi, R., & Prihandini, R. M. (2024). On (a, d)-hyperedge antimagic labeling of certain classes of hypergraphs: A new notion. *2nd International Conference on Neural Networks and Machine Learning 2023 (ICNNML 2023),* *2*(1), 173-183. https://doi.org/10.2991/978-94-6463-445-7_18

Dafik, Miller, M., Ryan, J., & Bača, M. (2009). On super (a, d)-edge-antimagic total labeling of disconnected graphs. *Discrete Mathematics*, *309*(15), 4909–4915. https://doi.org/10.1016/j.disc.2008.04.031

Dasar, K., Dan, H., & Wardayani, A. (2020). Konsep dasar hipergraf dan sifat-sifatnya [Basic concepts of hypergraphs and their properties]. *Jurnal Ilmiah Matematika dan Pendidikan Matematika,* *12*(2), 49-62. https://doi.org/10.20884/1.jmp.2020.12.2.3619

Hartsfield, N., & Ringel, G. (1990). *Pearls in graph theory*. https://books.google.co.id/books?hl=id&lr=&id=VMjDAgAAQBAJ&oi=fnd&pg=PP1&dq=Hartsfield,+N.,+and+Ringel,+G.+(1990).+Pearls+in+Graph+Theory. Academic Press&ots=AGLClgEeE_&sig=EYTa3CnihYw16J2Tasb0ohPALUQ&redir_esc=y#v=on

epage&q&f=false

Muhammad, J. (2013). Labeling of Graphs and Hypergraphs. *National University of Computer and Emerging Sciences Karachi*.

Muthuselvi, N., & Devi, T. S. (2025). Local super (a, d) edge antimagic total labeling of graphs. *IAENG International Journal of Applied Mathematics, 55*(1), 254-259. https://www.iaeng.org/IJAM/issues_v55/issue_1/IJAM_55_1_28.pdf

Nadzima, U., & Martini, T. S. (2019). Super (a, d)-H-antimagic total labeling on wheel edge corona product with a path and a cycle. *Journal of Physics: Conference Series, 1306*(1), 1-6. https://doi.org/10.1088/1742-6596/1306/1/012007

Parag, T., & Elgammal, A. (2011). Supervised hypergraph labeling. *In CVPR 2011, 2011*(1), 2289-2296). https://doi.org/10.1109/CVPR.2011.5995522

Prihandini, R., & Adawiyah, R. (2022). On super (*a*, *d*)-edge antimagic total labeling of some generalized shackle of fan graph. *International Journal of Academic and Applied Research,* *6*(4), 28-32. https://repository.unej.ac.id/xmlui/handle/123456789/111593

Saibulla, A., & Pushpam, P. R. L. (2025). On e-super (a, d) -edge antimagic total labeling of total graphs of paths and cycles. *Communications in Combinatorics and Optimization, 10*(4), 787-802. https://doi.org/10.22049/cco.2024.28592.1625

Series, C. (2016). The connected and disjoint union of semi jahangir graphs admit a cycle-super (*a*, *d*)-atimagic total labeling. *Journal of Physics: Conference Series, 693* (1), 1-7. https://doi.org/10.1088/1742-6596/693/1/012006

Smita, B. (2021). (Super)(a, d) - H- Antimagic total labeling of super sub division of cycle. *Arya Bhatta Journal of Mathematics and Informatics, 13*(2), 275-290.

Sonntag, M. (2002). Antimagic vertex labelings of hypergraphs. *Discrete Mathematics*, *247*(1–3), 187–199. https://doi.org/10.1016/S0012-365X(01)00175-3

Sumarno, D., Dafik, & Santoso, A. (2015). Super (a, d)-Edge Antimagic Total Labeling of Connected Ferris Wheel Graph. *Jurnal Ilmu Dasar, 15*(2), 123-130. https://doi.org/10.19184/jid.v15i2.1051

Tuczy, M. (2019). *On cordial labeling of hypertrees*. *21*, 1–14.

Venkatraman, S., Rajaram, G., & Krithivasan, K. (2018). Unimodular hypergraph for DNA sequencing: A polynomial time algorithm. *Proceedings of the National Academy of Sciences, India Section A: Physical Sciences, 90*(1), 49-56. https://doi.org/10.1007/s40010-018-0561-z

You, A., Be, M., & In, I. (2018). On super local antimagic total edge coloring of some wheel related graphs. *AIP Conference Proceedings, 2014*(1), 1-7. https://doi.org/10.1063/1.5054492