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The high-accuracy geometric approximation of the ellipse's perimeter by the measuring right-angled triangle

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***Abstract*:**
This article confronts the persistent challenge of determining the exact perimeter of an ellipse. It proposes a high-accuracy geometric approximation centred on a uniquely defined Measuring Right-Angled Triangle (MRAT). Constructed with specific spatial and angular properties, the MRAT is positioned at a distance of $2b/\pi$ from the centre of a reference circle and terminates at its circumference at a 45° angle. The ellipse's center is co-located with the circle's center. The resulting values were rigorously compared against classical Ramanujan approximations, and the PRI test and high-precision graphical analysis were used to confirm significant accuracy. This high-accuracy geometric approximation method offers a computationally efficient alternative to traditional algebraic methods, enhancing both theoretical understanding and applied precision.

Keywords: Conic Sections; Ellipse Perimeter; Geometric Approximation; Ramanujan's Approximations.

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Introduction

An ellipse is a plane curve defined by two principal axes: the central axis and the minor axis. The perimeter of an ellipse refers to the total length of the continuous boundary enclosing the shape. However, unlike circles or polygons, there is no closed-form expression for calculating the exact perimeter of an ellipse or any other figure among the conic sections using elementary functions. This longstanding mathematical problem has captivated scholars for centuries (Giacomoni, Pawan and Sreenadh, 2016; X. Yu, 2012). Historically, its origins can be traced to ancient Greek mathematicians such as Archimedes and Ptolemy (Al-Ossmi, 2023; Uralde-Guinea *et al*., 2026), who developed early approximation methods. Although these efforts lacked precision, they laid the groundwork for future advancements (David Chung and Richard Wolfgramm, 2015).

In the 17th century, the German astronomer and mathematician Johannes Kepler (Abbott, 2009; Ahmedi, 2018; Qureshi, Akhtar, & Ahamad, 2020) introduced a new geometric approach by relating elliptical shapes to planetary motion, thereby deepening the theoretical understanding of ellipses. Nevertheless, despite such progress, the precise evaluation of an ellipse's perimeter remained unresolved. Later, in the 18th century, mathematicians such as Leonhard Euler (Qureshi, Akhtar, & Ahamad, 2020) and Abraham de Moivre (Rohman, 2022) contributed improved approximations through infinite series and emerging tools in mathematical analysis. While these methods enhanced accuracy, they remained computationally intensive and analytically complex (Zhao & Chu, 2022). Subsequently, in the 19th century, mathematicians such as Carl Friedrich Gauss (Al-Ossmi, 2022; Asaad, Ahmed, & Ebrahim, 2022) and Bernhard Riemann further advanced the field by introducing elliptic integrals, yielding highly accurate but algebraically intricate solutions (Wang, Chu, & Chu, 2021). Over time, numerous approximation formulas have been proposed to estimate the perimeter of an ellipse. These include classical series expansions, Ramanujan's elegant approximations, and expressions derived from integral calculus. In general, three main approximation formulas are commonly used, depending on the ellipse's eccentricity. The first is suitable when the semi-axes a and b are nearly equal, approximating a circular shape. In contrast, the second and third are employed when there is a substantial difference between the two axes (Zhao, Wang, & Chu, 2022). Among these, one of the most significant contributions came from the Indian mathematician Srinivasa Ramanujan in 1923, who introduced formulas that produced remarkably accurate approximations of the ellipse's perimeter (Asaad, Ahmed, & Ebrahim, 2022). Ramanujan's formulas for estimating the perimeter of an ellipse gained wide recognition due to their simplicity, ease of use, and surprisingly high accuracy compared to more complex methods (Koshy, 2023; Wang, Chu, & Chu, 2021).

In recent decades, significant developments have emerged through the work of various scholars, including Barnard et al. (2001), Abbott (2009), and, more recently, Qureshi et al. (2020) and Al-Ossmi (2023 and 2024). Collectively, these studies reflect the ongoing interest in developing more precise, computationally feasible methods for evaluating the perimeter of an ellipse. Specifically, in 2023, Al-Ossmi introduced a novel geometric approach. This method is based on a high-accuracy geometric approximation using a Measuring Right-Angled Triangle (MRAT), which is positioned at a fixed distance of $2b/\pi$ from the centre of a reference circle (where b is the semi-major axis). Furthermore, the triangle terminates at a point on the circle's circumference, forming a 45° angle. Central to this construction is a special "Measuring Curve" (Appendix A), which determines the intersection point with the ellipse. The distance from this point to the ellipse's center defines the radius of a "Measuring Circle." Al-Ossmi consequently proved that the circumference of this Measuring Circle provides the exact value of the ellipse's perimeter.

Despite notable advancements in algebraic geometry, a pressing need persists for innovative geometric methods that offer greater simplicity and greater precision in solving classical problems, such as determining the perimeter of an ellipse. This paper addresses this need by further developing Al-Ossmi's method (2023). Specifically, we replace the original Measuring Curve with a straight-line approximation embedded within a right-angled triangle. This critical revision leads to a new geometric construction from which the exact perimeter of the ellipse can be derived. Notably, the proposed method, termed the Measuring Right-Angled Triangle (MRAT), remains valid for all variations of the ellipse's semi-major and semi-minor axes. The MRAT framework provides a high-accuracy geometric approximation, enabling the determination of the elliptical perimeter without reliance on complex algebraic expressions. Consequently, it significantly simplifies the calculation process while maintaining high accuracy. The core of this method involves constructing a unique "measurement circle" for each ellipse, whose circumference matches the ellipse's actual perimeter. Thus, this work offers a practical, geometrically intuitive, and analytically sound alternative to traditional perimeter-estimation techniques.

Research Methods

Using specialist mathematical software, the perimeter of an ellipse can be calculated to arbitrary precision. The value of various simpler approximations in this paper is that a geometric approach has adopted a high-accuracy geometric approximation of the Ellipse's Perimeter based on the properties of the elliptic curve, aiming to derive a new method intrinsically linked to the elliptic curve's geometric properties.

This study employs a high-accuracy geometric approximation to further develop Al-Ossmi's method (2023) for determining the exact perimeter of an ellipse. The original Measuring Curve, a central component of the previous method, were replaced by a right-angled triangle configuration. This transformation yields a new geometric structure, the Measuring Right-Angled Triangle (MRAT), designed to simplify and improve the accuracy of elliptical arc length measurements.

The research approach is structured around the following steps:

(1) Geometric Substitution: The curved segment previously used to approximate the elliptical arc is replaced by a right-angled triangle, whose dimensions are governed by the ellipse's semi-major and semi-minor axes. This substitution preserves geometric fidelity while enabling a simpler analytical framework.

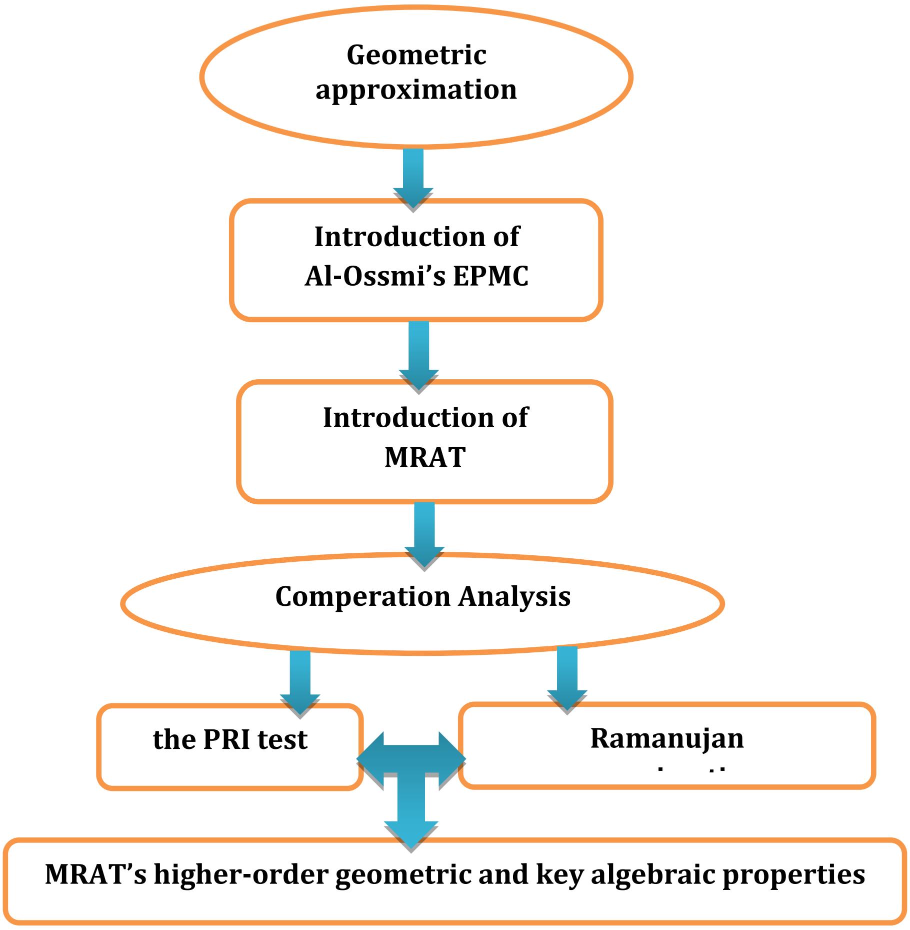
(2) Construction of MRAT: For each ellipse analyzed, a distinct right-angled triangle is constructed such that one of its sides corresponds to the arc segment under consideration. This triangle is positioned to geometrically represent the elliptical curvature by linear approximation, without resorting to algebraic elliptic integrals.

(3) Introduction of Measurement Circle: A novel concept introduced in this study is the *Measurement Circle*, a geometric construct uniquely associated with each ellipse. The circumference of this circle is defined to equal the exact perimeter of the ellipse, serving as a geometric proxy for perimeter estimation. The MRAT construction aids in analytically determining the dimensions of this circle.

(4) Generalization: The method is validated across a range of ellipses with varying semi-major and semi-minor axes. The robustness of the MRAT approach is confirmed by its applicability across a range of axial proportions.

(5) Comparative Analysis: The results obtained using the MRAT technique are compared with Ramanujan's classical approximations and further confirmed through the PRI test and high-precision graphical renderings, demonstrating significant accuracy.

By adopting this method, the study bypasses the need for complex integral calculus traditionally used in elliptical geometry, offering a visually intuitive and mathematically rigorous alternative. The MRAT framework represents a significant advancement in geometric methods for conic section analysis, particularly by simplifying perimeter determination while preserving precision (Flowchart 1).



Flowchart 1: the geometric workflow for ellipse perimeter approximation via the MRAT.

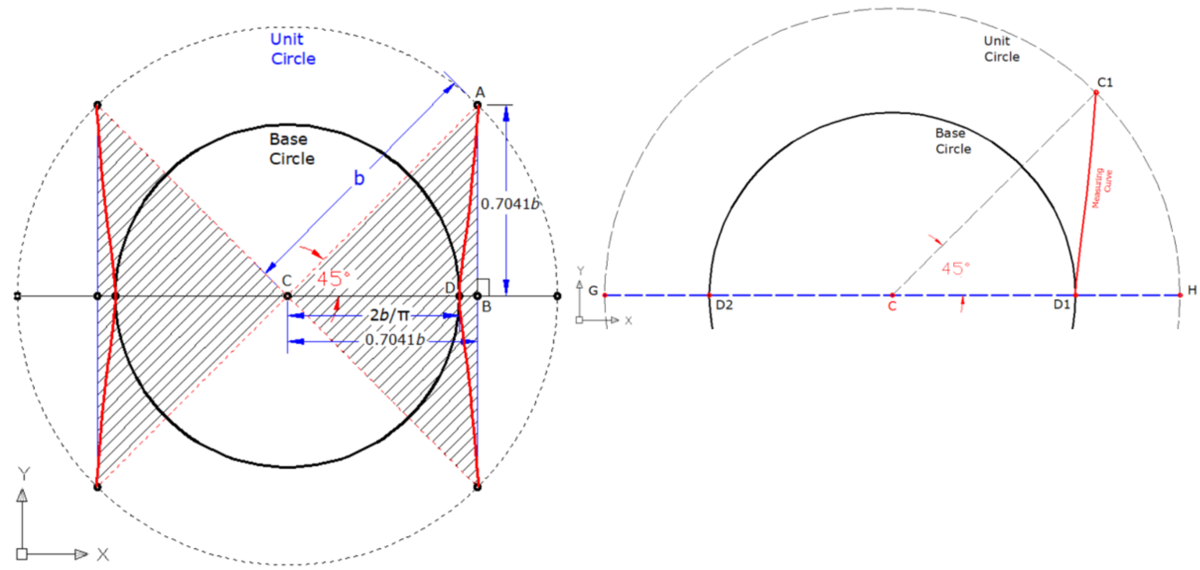
Al-Ossmi's Ellipse Perimeter Measuring Curve (EPMC)

Al-Ossmi's EPMC (2023) introduced a geometric approach grounded in two-dimensional geometry, utilizing the lengths of the ellipse's semi-major axis (a) and semi-minor axis (b) to determine its perimeter. In this method, the center of the ellipse (C) is at the origin (0, 0), with the foci aligned along the x-axis and the minor axis along the y-axis. As presented in Al-Ossmi's EPMC, the circumference of the larger, outer circle determines the location of the highest point on the Measuring Curve. In contrast, the smaller, inner circle defines the endpoint of the curve. Importantly, the radius of the inner circle is constant and represents a new geometric parameter introduced in this study, termed the Base Circle constant. In contrast, the radius of the outer circle varies depending on the length of the central axis of the specific ellipse being studied. It is evident that the radius of the Base Circle (denoted as CD_1) establishes the minimum possible value for the radius of the Measuring Circle ($CB = r$). This minimum occurs in the limiting case where the semi-minor axis is reduced to zero ($a = 0$), resulting in a degenerate ellipse. Under this condition, the Measuring Circle and the Base Circle become identical, sharing the same radius $CD_1 = \pi 2b$. Consequently, the resulting curve is no longer elliptical but instead becomes a straight-line segment GH with length $2b$ (Picture 1). Notably, the radius of the Base Circle is a constant and can be expressed as:

$$CD_1 = \left(\frac{2b}{\pi}\right) = 0.63661977236758134307553505349006\, b, \tag{1}$$

$$D_1H = b - \left(\frac{2b}{\pi}\right) = 0.36338022763241865692446494650994\, b, \tag{2}$$

Building on these geometrical ratios, Al-Ossmi's EPMC method consisted of a key set of geometry properties related to the ellipse. Accordingly, based on the standard definition of an ellipse, Figure 1 illustrates a representative configuration of $\left(\frac{2b}{\pi}\right)$, where a and b denote the semi-major and semi-minor axes, respectively.

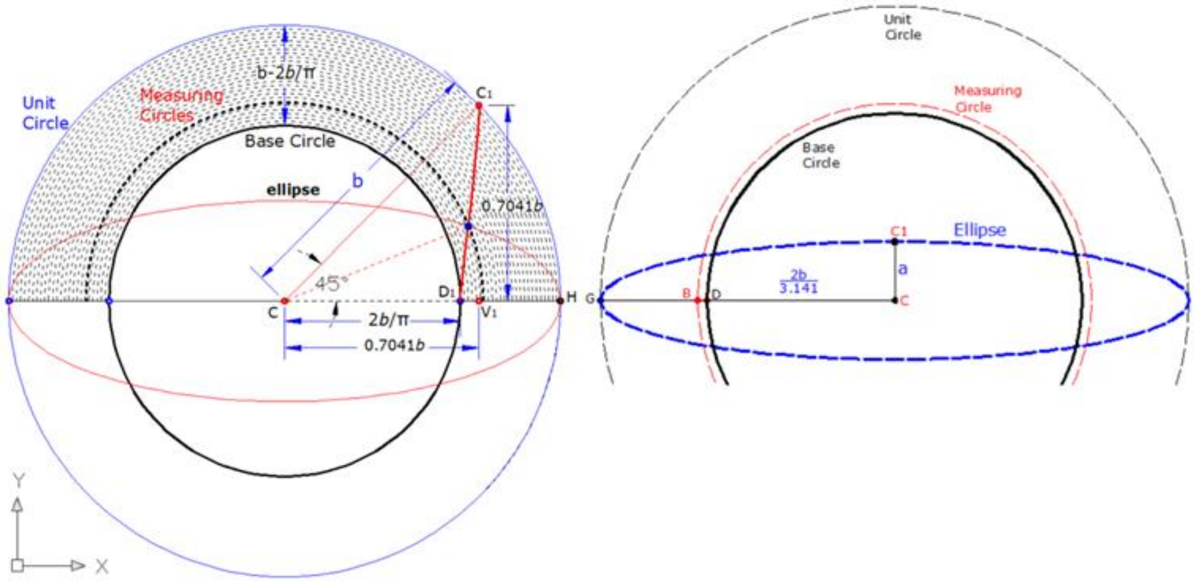


Picture 1. Al-Ossmi's EPMC method consisted of a key set of geometry properties, where the Base Circle with radius of $\left(\frac{2b}{\pi}\right)$, and the Unit Circle with radius of (b), (see Al-Ossmi, 2023).

Based on the described construction steps (Picture 1), the Base Circle has a radius denoted as CD_1shares the same center as the ellipse. According to the geometric proportions of the ellipse, three distinct curvature cases can be identified, one of which degenerates into a straight line. The EPMC's method is designed to accommodate all these possible cases: an ellipse, a circle, and a line. As the value of the semi-major axis, b, varied from zero to b, the resulting curve transitions accordingly from a straight line to an ellipse and, eventually, to a circle. Also, Al-Ossmi's EPMC method (2023) consisted of a key set of geometry cases categorized as follows:

1. When a = 0: The ellipse collapses into a straight-line segment of length $4b$.
2. When a = b: The ellipse becomes a perfect circle with radius b, and its circumference is equal to EP/b, where EP denotes the ellipse perimeter in this configuration.
3. When a < b: The resulting figure remains an ellipse, and its perimeter can be approximated by the circumference of the corresponding Measuring Circle, expressed as 2πr, where r is the radius determined geometrically through the EPMC's method.

This classification demonstrates the EPMC's flexibility in addressing a wide range of elliptical forms, from degenerate to circular, by varying the semi-major axis within the defined bounds. Focusing on the determination of the radius of the Measuring Circle, this radius (r) is pivotal in EPMC's Method, utilized for the precise computation of the perimeter of the ellipse. The Measuring Circle is a unique circle associated with each instance of an ellipse, and there is only one Measuring Circle per segment scenario. Its circumference equals the perimeter of the ellipse segment that shares the exact center point and intersects its perimeter at the same point along the Measuring Curve. In the definition of the EPMC, the Measuring Circle is confined within the space defined by the Base Circle and the Unit Circle. When it lies on the Base Circle, the result is a straight line along the x-axis, whereas if it lies on the Unit Circle, the result is a circle rather than an ellipse segment (as illustrated in Picture 2).



Picture 2. Within the EPMC, the Measuring Circle is geometrically constrained to the region bounded by the Base Circle and the Unit Circle, ensuring its radius remains within defined limits throughout the construction process.

According to Al-Ossmi (2023), the validity of the EPMC is established through the geometric representation of the elliptical segment, projected from the common centre shared by the Base Circle and the Unit Circle within the EPMC framework (Picture 2). The method introduces several key properties associated with the Measuring Circle:

(1) Each elliptical segment corresponds to a unique Measuring Circle.
(2) The Measuring Circle must pass through the center of the ellipse under investigation.
(3) The intersection point between the ellipse and the Measuring Curve defines the terminal point of the radius of the Measuring Circle, which shares a common center with the ellipse.
(4) As a result, the Measuring Circle is uniquely associated with the studied elliptical segment, and its circumference is equivalent to the exact perimeter of that segment.
(5) By applying the standard formula for the circumference of a circle, the perimeter of the ellipse segment can be calculated algebraically, thereby fulfilling the main objective of this research.

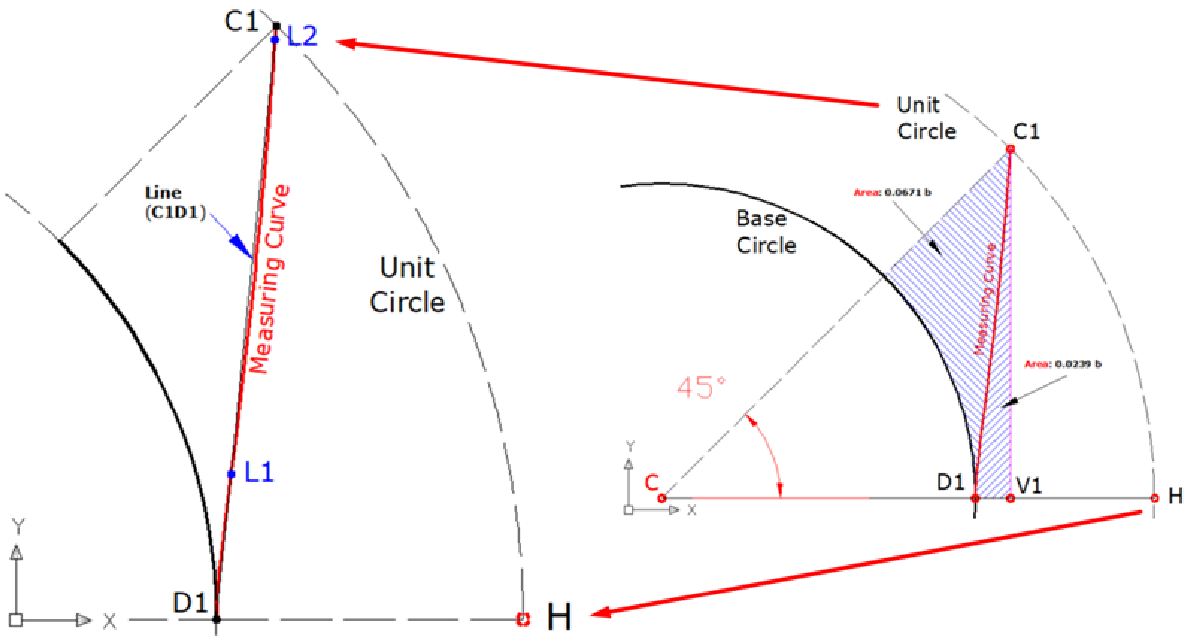
These geometric properties will be formally demonstrated and validated in the Measurement section of the study. Furthermore, the proposed method will be rigorously tested across multiple elliptical segments and compared with existing results, particularly those derived from Ramanujan's approximation formulas. According to the properties of the EPMC, the Measuring Curve is constructed by first drawing both the Unit Circle and the Base Circle, which share a common center point, denoted as C. From this central point, a ray is projected at an angle of 45°, intersecting the Unit Circle at point A. From point A, a perpendicular segment of length 0.7041b is dropped to intersect the x-axis at point D, forming a right-angled isosceles triangle △ABC, where the hypotenuse corresponds to the precisely length of the central axis of the ellipse, denoted as b (Picture 3).

It is important to note that this triangle can be symmetrically represented in all four quadrants of the Unit Circle due to the geometric symmetry of the construction. Within the EPMC framework, the Measuring Curvature, labelled. C_1D_1, is treated as a constant geometric feature. It includes three inflexion points (or points of twisting) and has a fixed arc length of approximately $0.7102b$, as detailed in Tables 1 and 2. Hence, two critical geometric properties emerge from the EPMC construction:

(1) The area enclosed by the Measuring Curve and the two segments C_1V_1 and D_1V_1 is constant, with a value equal to $0.02390b$.

(2) The area bounded by the Measuring Curve, the segment CC_1, and the arc of the Base Circle is also constant, with a value equal to $0.06710b$.

Also, Picture 3 detailed the key features of a right-angled triangle. (ΔCC_1V_1), where the Measuring Curve is delineated by the Base Circle and the Unit Circle in the definition of the EPMC.



Picture 3. Plot of the key features of a right-angled triangle (ΔCC_1V_1) where the Measuring Curve is delineated by the Base Circle and the Unit Circle in the definition of the EPMC (see Appendix A).

Table 1. The Measuring Curvature's Properties in EPMC

Code	Value	Type	Description
B_2H	٠,٧٠٦٥b	Line	($B_2H = CV_1$), if the angle (B2CH) $= \left(\frac{\pi}{2}\right)$
CC_1	b	Line	b
C_1H	٠,٧٦٤٥b	Line	Constant
C_1V_1	٠,٧٠٦٥b	Line	$b \sin 45$
D_1V_1	٠,٠٧١١b	Line	$b \cos 45 - \left(\frac{2b}{\pi}\right)$
CD_1	٠,٦٣٦٦b	Line	Constant = $\left(\frac{2b}{\pi}\right)$
C_1D_1	٠,٧١٠٢b	Curve	Measuring Curvature (Constant)
CH	b	Line	b
CV_1	٠,٧٠٧٧b	Line	Constant $= b \cos\left(\frac{\pi}{4}\right)$
V_1H	٠,٢٩٢٣b	Line	Constant
C_1CH	٤٥	Angle	Constant
CB	$CD_1 \geq CB \leq b$	Line	$\left(\frac{2b}{\pi}\right) \geq r \leq b$

Where:

(a) It is the semi-minor axis of the ellipse.

(b) It is the semi-major axis of the ellipse.

(r) is the radius of the Measuring Circle, BC.

All detailed figures and quantitative results of all conducted cases are listed in Table A, Appendix A.

Table 2. Measuring Curvature's Key Properties in EPMC

Code	Measuring Curve	Line
(C_1L_2)	0.01697449 b	0.01697420 b
ratio	1.00001708475215326790	
Code	Measuring Curve:	Line:
(L_1L_2)	0.5194 b	0.5193 b
ratio	1.00019256691700300000	
Code	Measuring Curve	Line
(D_1L_1)	0.1739 b	0.1738 b
ratio	1.00057537399309551208	
Code	Measuring Curve	Line
($D_1L_1L_2C_1$)	0.71023974 b	0.71005633 b
ratio	1.0002583034503755497820	

Built from figures 1, 2, and 3.

Where:

(a) It is the semi-minor axis of the ellipse.

(b) It is the semi-major axis of the ellipse.

(r) is the radius of the Measuring Circle, BC.

According to the MRAT definition (Al-Ossmi, 2023), the ellipse parameters are consistent with the ratio a/b, "the semi- minor/major axis"; hence, they are determined by the intersection point of the ellipse with the Measuring Curve. Also, as seen in equations 1 and 2, table 2 shows that if a = 0, then the curvature becomes a straight line of length 2b, and the perimeter limit = 4b (classical limit). However, the parameter result is if a = b, since the curvature is a circle of radius a, then P = 2πa. All these properties are taken into account to transform the EPMC and its action into MRAT.

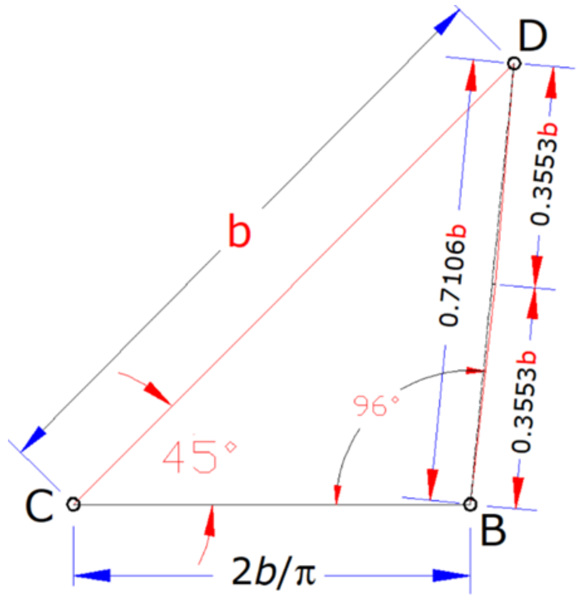
The MRAT Developed Method:

The Measuring Triangle (MT)

In this stage, Al-Ossmi's EPMC is represented by developing the Measuring Right-Angled Triangle (MRAT), replacing the Measuring Curve by the Measuring Triangle, since in the framework of the EPMC, the Measuring Circle is geometrically constrained within the range of $\left(\frac{2b}{\pi}\right)$, bounded by the Base Circle and the Unit Circle, while the ratio of a/b is varied, both key circles' radii remain within defined limits throughout the construction process, Fig. 4.

The key point at this stage is that the purpose of adding the Measuring Triangle is that this triangle serves to simplify complex curved measurements by:

- Replacing the nonlinear path of the Measuring Curve (see Appendix A) with a geometric approximation that is analytically simpler.
- Allowing for direct trigonometric calculation using known angles and side ratios.
- Enabling designers or analysts to convert continuous curvature into a discrete geometric form with clearly defined relationships (lengths and angles).



Picture 4. Plot the MRAT key geometrical proportions of the Measuring Triangle with a precise angle of 96 degrees. Where C is the ellipse center, and $CD = b$, the ellipse semi-major axis, and then a fixed ratio of $\left(\frac{2b}{\pi}\right)$ Presents the radius of the Unit Circle (see Appendix A).

MRAT Geometrical Proportions

Based on the provided transformation, a description of the algebraic concept and geometric properties involving the EPMC and its transformation into MRAT by adding a straight line within the Measuring Triangle (MT). In this stage, the EPMC is approximated geometrically as a straight line, transforming a previously curved measurement path into a linear, analyzable segment. A right-angled triangle represents this transformation, termed the MT, in Picture 4 (Appendix A).

Key Elements of MT can be listed as:

(1) The right-angled triangle $\triangle CDB$ is the Measuring Triangle.
(2) Side $CD = b$ is the hypotenuse, which represents the MRAT's linear approximation.
(3) Then, the angle at point C is 45°.
(4) The angle at point B is labelled as 96°, indicating the inclination of the vertical projection.

All presented ratios were produced according to the right-angled triangle △CDB (Picture 4). From the right-angled triangle △CDB, the horizontal side CB is defined as:

$$CB = \frac{2b}{\pi}, \quad (3)$$

This specific ratio $(2b/\pi)$ is produced by the EPMC. At this stage, it is used in MT since it suggests substituting arc length with its linear equivalent based on circular geometry, where the chord approximates the arc.

The transformation of the Measuring curve of EPMC into a straight-line approximation represented by a right-angled Measuring Triangle $\triangle CDB$, Picture 4, has been used to produce the following components, where:

(1) The hypotenuse $CD = b$ replaces the EPMC's measuring curve path.
(2) The horizontal leg $CB = \pi 2b$ approximates arc length as a chord.
(3) The vertical segment is a constant, where $DB = 0.7106b$, it is symmetrically split, aiding analysis.
(4) This stage facilitates further calculation and understanding by translating the EPMC's measuring curve into structured linear geometry.

These ratios are used in this stage as spatial measurements derived from the trigonometry rules of △CDB and as ratios constructed proportionally to the base b, implying that the height characterises accumulated effects along the EPMC's measuring curve (e.g., slope, displacement, or other spatial measurements).

Results and Discussions

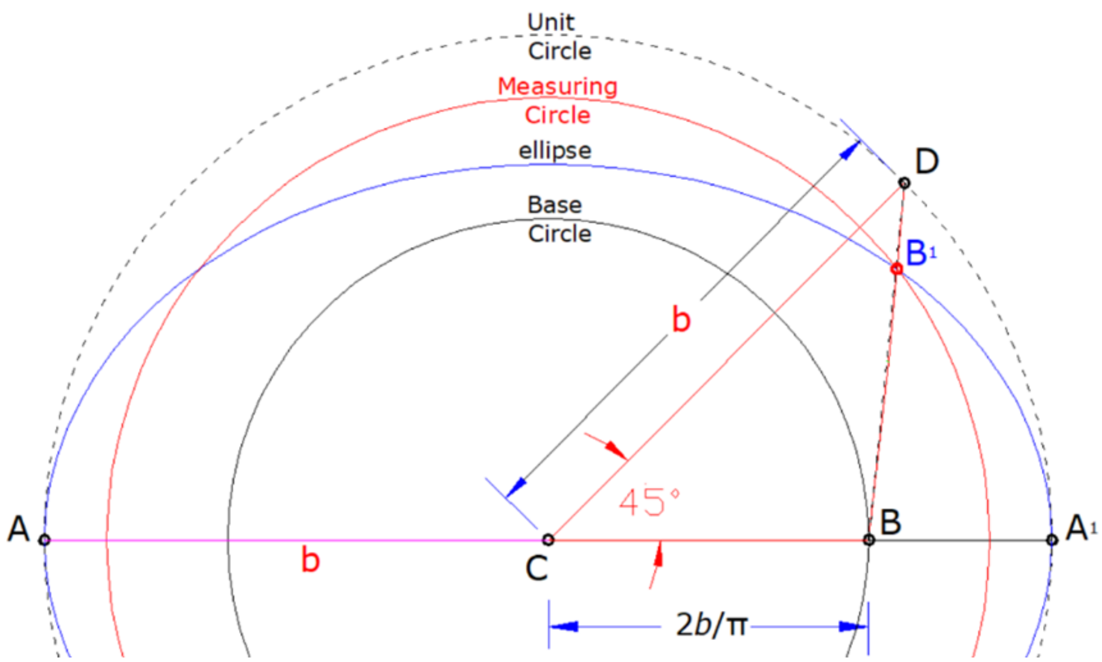
Practical Examples

The geometric foundation of the innovative method proposed in this study is applied, MRAT. The core idea relies on the geometric ratios obtained from the right-angled triangle $\triangle CDB$, which constitutes the fundamental geometric and mathematical basis of the approach as a whole. In this section, the variables represent these ratios between the Unit Circle and the Base Circle that are linked with the ratio of major and minor axes of the ellipse, whose values vary according to the specific configuration of the ellipse examined in each case. In this stage, a set of 12 varied ellipses was used to test the MRAT method. All cases presented as representative standard examples intended to calculate the ellipse perimeter values produced by this method, tested and can be consulted in the study's appendix (A).

Since (a) and (b) are the semi-major and semi-minor axes, respectively, applying the MRAT method included steps that are listed as follows, Picture 5:

(1) Let an ellipse whose semi-major axis is b, and a semi-minor axis is a, with a center point as C.
(2) Then, draw the Unit Circle with a radius equal to the length of the major axis of the ellipse, b.
(3) From the center point C of the Unit Circle, draw the Base Circle with a specified radius $\left(\frac{2b}{\pi}\right)$.
(4) Construct the Measuring Curve according to the geometric specifications defined in the EPMC. In MRAT, this involves drawing a ray at a 45º angle from the Unit Circle centre, C, to intersect the Unit Circle, then dropping a perpendicular from the intersection point to form the necessary right-angled isosceles triangle.
(5) Finally, plot the ellipse whose perimeter is to be calculated. Ensure that this ellipse is aligned with the horizontal diameter of the Unit Circle and shares the same center point C as the Unit Circle.
(6) The intersection point between the ellipse and the BD segment is B_1.
(7) Draw the ray (CB_1), which is the radius of the Measuring Circle.
(8) The circumference of the Measuring Circle gives the exact value of the ellipse perimeter.

By following these steps, we can now achieve the main goal of this paper: visually and geometrically determine the necessary parameters for calculating the ellipse's perimeter using the EPMC described in this study (Picture 5).



Picture 5. Visualisation of geometrical elements of the MRAT method, to determine the highly accurate value of the ellipse perimeter.

In this stage, a set of 12 varied ellipses was used to test the MRAT method. By fixing the semi-major axis of the ellipse, b=1, the ellipse semi-miner axis varied from a = 1.01 to (a = b), hence the Measuring Circle's radius varied till; $\left(CB = \frac{2b}{\pi}\right)$. According to the MRAT, the circle drawn with a radius of $\frac{2b}{\pi}$, while the Unit circle is drawn with radius b. The obtained calculations are illustrated and listed, see Appendix A.

In the MRAT, by fixing the value of the semi-minor axis $0 \geq a \leq b$, then an ellipse is randomly nominated with a semi-minor axis, then by the MRAT method definition and its MT, the Measuring Circle's radius is geometrically calculated to give the circumference's value, which is the ellipse's perimeter, as in Table 3. Because the MRAT is an approximation method built on geometric proportions, comparisons are made against a high-precision approximation value for Ramanujan's approximation method. The data in Table 3 present a quantitative comparison between the MRAT method and Ramanujan's approximation method for calculating the perimeter of a set of ellipses. The key variables show that the MRAT method demonstrates excellent agreement with the well-established Ramanujan's approximation across the full range of elliptical forms. Given its geometric approximation, the MRAT approach provides not only numerical accuracy but also a visual and constructible method for perimeter estimation. The small, bounded absolute error indicates that EPMC is a reliable and precise alternative to algebraic approximations, especially useful in contexts where geometric constructions are preferred or required (Table 3).

Table 3. Comparison of MRAT 's results by Ramanujan's method.

a	MRAT 's	Ramanujan's	Absolut Error
0.000	$4b$	$4b$	0.00000000
0.01 b	4.00102262 b	4.00109915b	0.00007653
0.1 b	4.06396426 b	4.06397418 b	0.00000992
0.2 b	4.20213150 b	4.20200891 b	-0.00012259
0.3 b	4.38589970 b	4.38591007 b	0.00001037
0.4 b	4.60255900 b	4.60262252 b	0.00006352
0.5 b	4.84419997 b	4.84422411 b	0.00002414
0.6 b	5.10539996 b	5.10539977 b	-0.00000019
0.7 b	5.38240009 b	5.38236898 b	-0.00003111
0.8 b	5.67229971 b	5.67233358 b	0.00003387
0.9 b	5.97320000 b	5.97316043 b	-0.00003957
b	6.28318531 b	6.28318531 b	0.00000000

Where (a) and (b) are the lengths of the semi-major and semi-minor axes, respectively. All detailed figures and quantitative results for all conducted cases are listed in Appendix A, Table A.

Analysis of Results

As mentioned previously, the MRAT method is a high-accuracy geometric approximation of the Ellipse's Perimeter using a Measuring Right-Angled Triangle. Therefore, all obtained results were compared against a high-precision reference value for Ramanujan's approximation for calculating the perimeter of an ellipse (Table 3).

Detailed figures and quantitative results for all conducted cases are listed in Appendix A. The key variables are:
(1) The semi-major axis normalized as a fraction of b (the semi-minor axis),
(2) MRAT's Result: ellipse perimeter calculated using the MT's proportions,
(3) Ramanujan's Results: Ramanujan's second approximation,
(4) Absolute Error: the difference between the MRAT –Ramanujan's.

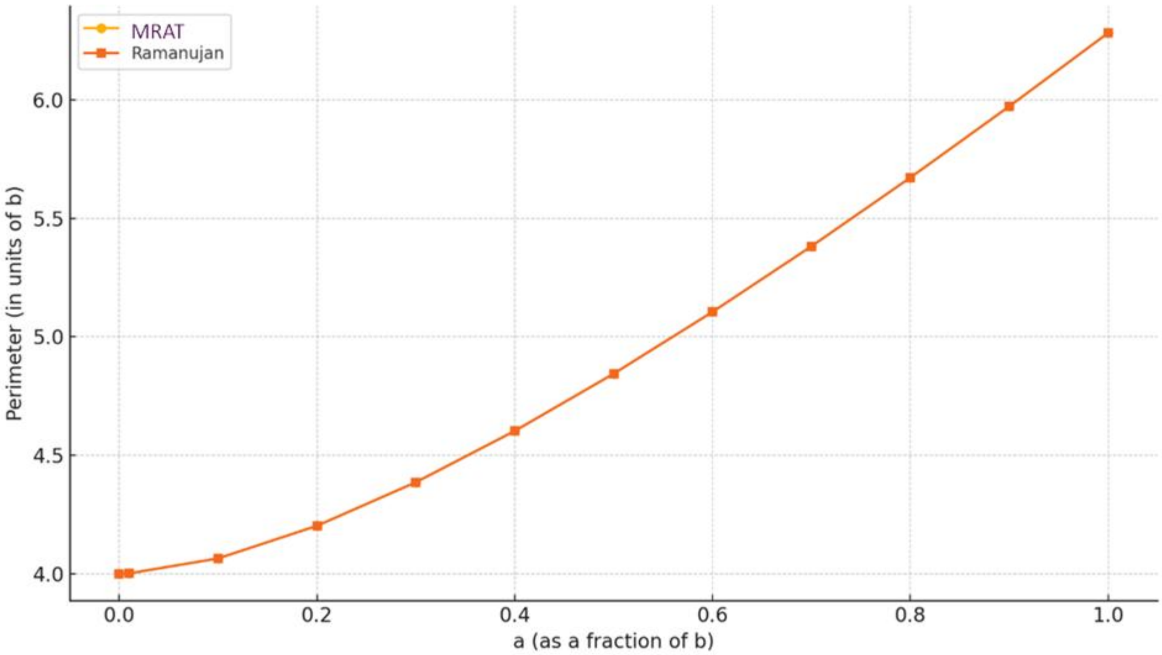
Accuracy & Agreement:
a. The MRAT method closely agrees with Ramanujan's method across the entire interval $0 \leq a \leq b$.
b. The absolute errors are all tiny, with high approximation on the order of 10^{-5} to 10^{-7}, indicating high numerical consistency.
c. At the endpoints, $a = 0$ and $a = b$, both methods give identical results (i.e., $4b$ and $2\pi b \approx 6.28318531b$), showing that both models reduce correctly to a line or a circle.

Symmetry & Consistency:
a. The absolute error fluctuates around zero, with both positive and negative values. It suggests that:
b. The variation is random and likely due to rounding and geometric constraints, not systematic bias.
c. Thus, the MRAT method is not consistently overestimating or underestimating.

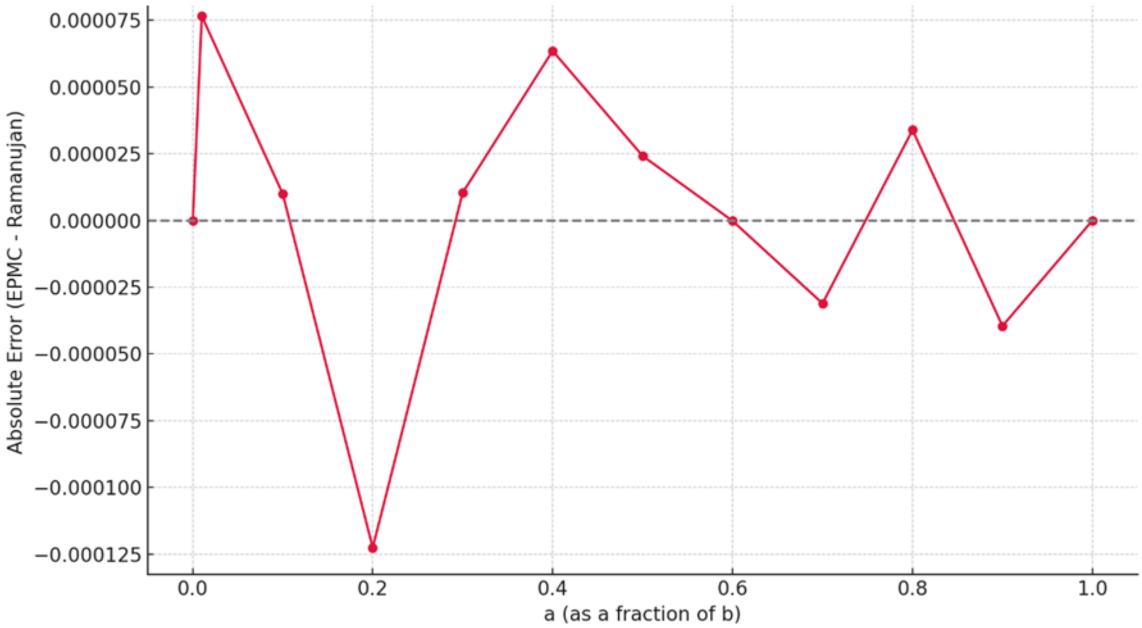
Trend with ratio a/b:
a. The maximum absolute error is calculated around $0.0003957b$ when $a = 0.9b$, which is still extremely small (~0.0066% relative error for a perimeter ≈ 6).
b. For $a = 0$ and $a = b$, the error is exactly zero, confirming perfect agreement at the endpoints (line and circle cases).
c. As the value of a increases from 0 to b, both methods predict an increasing perimeter, as expected, since the ellipse becomes more circular and its perimeter grows.

For additional validation, the PRI test was used at this stage. The Priority Rating Index is a numerical scoring method used to rank items, variables, or cases based on specific criteria. The PRI test is not a single, universally defined statistical test, but rather a set of tests used in many research contexts, especially in engineering, environmental studies, and decision analysis. PRI is commonly used as an abbreviation that can be easily visualised in charts (bar, radar, comparative plots, etc.). In this paper, the efficiency of MRATs by Ramanujan's method was compared using the PRI test to visualise the accuracy of these approximating methods, where each variable is assigned a score based on predefined criteria. The obtained results are shown in Pictures 6 and 7.

Results from Picture 6 indicate that both curves follow an almost identical trend, confirming the high accuracy of the MRAT's method. The two methods produce visually indistinguishable results across all values of a, indicating excellent numerical agreement. Compared with EPMC, the MRAT method effectively replicates Ramanujan's outcomes using a purely geometric construction. Evidence showed that the error remains very close to zero across all values of a, reinforcing the high precision of the MRAT method.



Picture 6. Comparison of MRAT's efficiency by Ramanujan's method using the PRI test. Where *a* is the semi-major axis (normalized as a fraction of *b*, the semi-minor axis).



Picture 7. Plot of Absolute Error compared with MRAT and Ramanujan's method. Where *a* is the semi-major axis (normalized as a fraction of *b*, the semi-minor axis).

Also, the Absolute Error values are compared with the MRAT and Ramanujan's method. Figure 7 shows that the maximum deviation occurs at $a = 0.2b$, with an error of approximately $-0.00012b$, which is still extremely small. The errors alternate in sign, showing no consistent bias, which supports the claim that MRAT is not systematically over- or underestimating the perimeter. Detailed figures and quantitative results for all conducted cases are listed in Appendix A.

The graphical analysis of the data confirms the accuracy, reliability, and consistency of the MRAT method for calculating the perimeter of ellipses. The first plot shows near-perfect alignment between the MRAT results and Ramanujan's established approximation for all values of the semi-major axis a, ranging from 0 to b. It suggests that the geometric MRAT method effectively captures the actual perimeter values with no visible deviation.

The second plot, illustrating the absolute error, further supports this conclusion. The error remains extremely small and well-distributed around zero, with no systematic bias toward overestimation or underestimation. The maximum observed error is less than 0.00013b, which is negligible in practical applications. Importantly, the error converges to zero at the boundary cases:

a. When a = 0, representing a straight line.
b. When a = b, representing a circle.

This validates that the MRAT method correctly handles the limiting cases of the ellipse and adapts smoothly across the full spectrum of elliptical shapes. The MRAT method offers a geometrically grounded, computationally efficient, and highly accurate alternative to traditional algebraic approximations, such as Ramanujan's formulas. Its performance, both visual and numerical, establishes it as a robust tool for theoretical and applied work in fields that require precise arc-length measurements, such as mathematics, engineering, astronomy, and computer graphics.

Discussion

1. High Numerical Consistency Across a/b Ratios:
   The Absolute-Error profile demonstrates that the MRAT method preserves a high degree of numerical consistency when compared with Ramanujan's approximation across the full range of a/b. The deviations remain confined to the order of 10^{-5} to 10^{-7}, indicating that the MRAT geometrical method is stable and insensitive to extreme geometric variations in ellipse shape.

2. Localized Error Amplification at Moderate Eccentricities:
   A pronounced negative deviation occurs near a/b ≈ 0.2, suggesting a localized sensitivity of the geometric method, MRAT, at moderate eccentricity levels. It reflects a region where slight variations in the geometric ratios of the ellipse's MRAT induce a slightly higher response. Nonetheless, the magnitude of this deviation remains minor and does not affect the overall reliability of the perimeter approximation.

3. Geometric Convergence at Specific Axis Ratios:
   The near-zero error recorded at a/b = 0,0.60, and 1.0 indicates apparent geometric convergence between the MRAT formulation and Ramanujan's classical expression. This approach utilizes three concentric circles with two fixed radii, b and π2b, where a represents the ellipse's minor axis (aligned with the y-axis), and b the central axis (aligned with the x-axis).

4. Four parameters govern the lengths of the Measuring Circle and the Measuring Triangle, which, in turn, measure roughly 0.71019548b. The Measuring Triangle in

MRAT encompasses an area defined by its vertical and horizontal projections, a constant $0.0239b^2$.

5. These points coincide with configurations in which the proportional relationships embedded in the elliptic-segment framework align particularly well with the curvature of the ellipse.

6. Absence of Systematic Overestimation or Underestimation:
The oscillatory pattern, with alternating positive and negative deviations, confirms that the method does not exhibit a systematic bias. The lack of directional drift in the error distribution indicates that the MRAT approach neither consistently overpredicts nor underpredicts the ellipse perimeter, an essential criterion for unbiased approximation methods.

7. Applicability to High-Precision Analytical and Computational Tasks:
The modified EPMC, MRAT, achieves an exceptional precision of approximately 2.4015×10^{-7} digits. Given its consistently low error levels and unbiased behaviour, the MRAT method demonstrates strong suitability for high-precision geometric modelling, computational analysis, and algorithmic applications. Its robustness across disparate a/b ratios supports its use as a dependable alternative formulation in both theoretical and applied mathematical contexts.

Conclusion

This paper presents a high-accuracy geometric approximation of the ellipse's perimeter using the Measuring Right-Angled Triangle (MRAT), modifying Al-Ossmi’s base method of the Ellipse Perimeter Measuring Curve (EPMC) to determine the perimeter of an ellipse accurately. The study introduces a new geometric construction for key features of the conic curve, enabling precise perimeter measurements. The method identifies four constants and introduces a novel form of special curvature. By fixing these constants, the MRAT can compute the exact circumference of ellipses across a wide range from flattened to inflated shapes. This research lays the groundwork for further investigations, such as exploring new algebraic modules and examining projective planes of conics, which may reveal richer geometric configurations. Additionally, a deeper study of the MRAT’s higher-order geometric and algebraic properties could uncover essential applications in mathematics and physics. Given its remarkable accuracy and conceptual simplicity, the MRAT holds promise for diverse practical applications in the future.

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Author's declaration

Conflicts of Interest: None. I hereby confirm that all the Figures and Tables in the manuscript are mine. Besides, the Figures and images, which are not mine, have been permitted republication and are attached to the manuscript.

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Appendix A

Table A. The Measuring Curve’s coordinators with an accuracy of 8 digits.

y	*x*	*y*	*x*	*y*	*x*
0.02510111	0.63738109	0.01755083	0.63700109	0.٠٠٠٠٠٠٠٠	0.63663569
0.02527245	0.63739109	0.01778667	0.63701109	0.00229474	0.63664109
0.02544279	0.63740109	0.01801961	0.63702109	0.00394998	0.63665109
0.02561215	0.63741109	0.01824976	0.63703109	0.00498347	0.63666109
0.02578057	0.63742109	0.01847725	0.63704109	0.00580513	0.63667109
0.02594805	0.63743109	0.01870216	0.63705109	0.00650935	0.63668109
0.02611461	0.63744109	0.01892459	0.63706109	0.00713615	0.63669109
0.02628028	0.63745109	0.01914462	0.63707109	0.00770695	0.63670109
0.02644506	0.63746109	0.01936235	٠,٦٣٧٠٨١٠٩	0.00823481	0.63671109
0.02660897	0.63747109	0.01957784	0.63709109	0.00872839	0.63672109
0.02677204	0.63748109	0.01979118	0.63710109	0.00919378	0.63673109
0.02693427	0.63749109	0.02000244	0.63711109	0.00963546	0.63674109
0.02709568	0.63750109	0.02021167	0.63712109	0.01005683	0.63675109
0.02725629	0.63751109	0.02041895	0.63713109	0.01046056	0.63676109
0.02741610	0.63752109	0.02062434	0.63714109	0.01084878	0.63677109
0.02757514	0.63753109	0.02082789	0.63715109	0.01122322	0.63678109
0.02773342	0.63754109	0.02102965	0.63716109	0.01158530	0.63679109
0.02789095	0.63755109	0.02122968	0.63717109	0.01193623	0.63680109
0.02804774	0.63756109	0.02142802	0.63718109	0.01227703	0.63681109
0.02820380	0.63757109	0.02162473	0.63719109	0.01260858	0.63682109
0.02835915	0.63758109	0.02181985	0.63720109	0.01293162	0.63683109
0.02851380	0.63759109	0.02201342	0.63721109	0.01324681	0.63684109
0.02866775	0.63760109	0.02220548	0.63722109	0.01355473	0.63685109
0.02882103	0.63761109	0.02239607	0.63723109	0.01385589	0.63686109
0.02897364	0.63762109	0.02258523	0.63724109	0.01415074	0.63687109
0.02912559	0.63763109	0.02277300	0.63725109	0.01443969	0.63688109
0.02927690	0.63764109	0.02295941	0.63726109	0.01472310	0.63689109
0.02942756	0.63765109	0.02314449	0.63727109	0.01500130	0.63690109
0.02957760	0.63766109	0.02332827	0.63728109	0.01527459	0.63691109
0.02972702	0.63767109	0.02351079	0.63729109	0.01554323	0.63692109
0.02987583	0.63768109	0.02369208	0.63730109	0.01580747	0.63693109
0.03002404	0.63769109	0.02387216	0.63731109	0.01606754	0.63694109
0.03017166	0.63770109	0.02405106	0.63732109	0.01632364	0.63695109
0.03031870	0.63771109	0.02422881	0.63733109	0.01657597	0.63696109
0.03046516	0.63772109	0.02440543	0.63734109	0.01682469	0.63697109
0.03061106	0.63773109	0.02475538	0.63736109	0.01706998	0.63698109
0.03075640	0.63774109	0.02492877	0.63737109	0.01731198	0.63699109

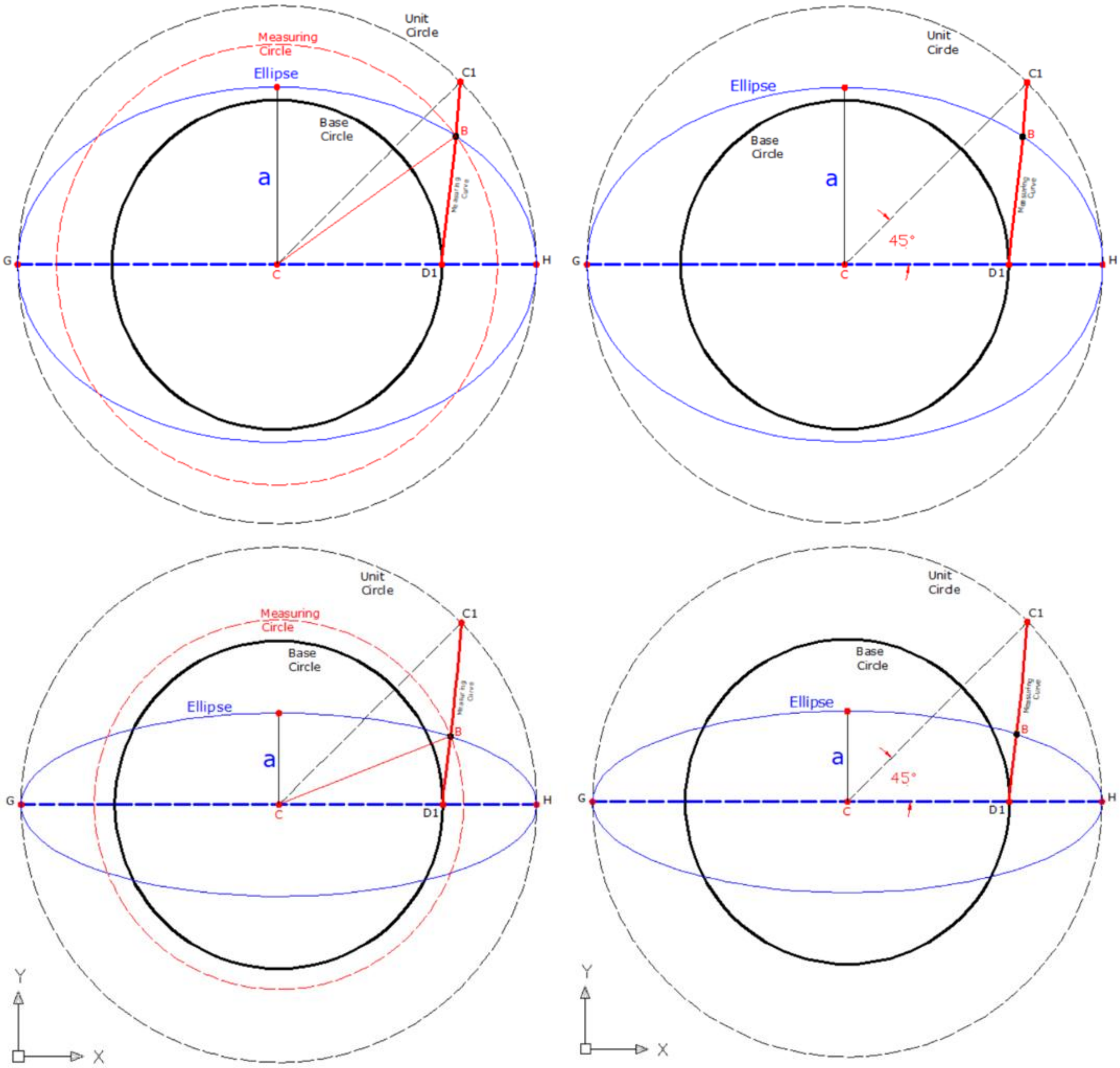

Figure A1. Applying the EPMC with an example of an ellipse, with value of (a = 0.68570692b), since the intersection point of the ellipse and the Measuring Curve is (B), then the Measuring Circle's radius is (CB = 0.85020428b).

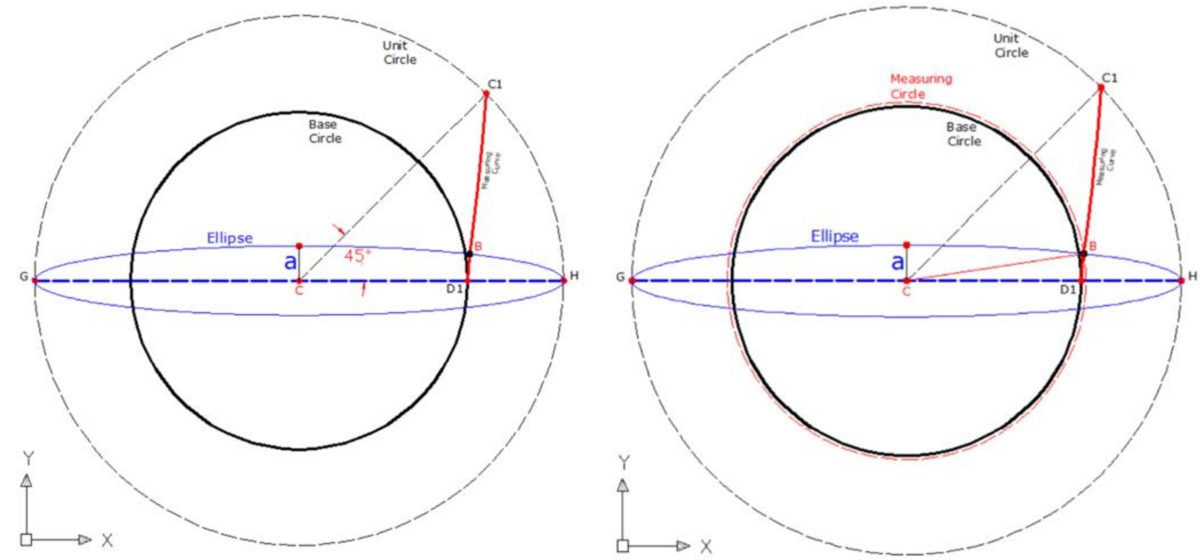

Figure A2. Applying the EPMC with an example of an ellipse, with a value of (a = 0.13080759b), and the Measuring Circle's radius is (CB = 0.91759019b).

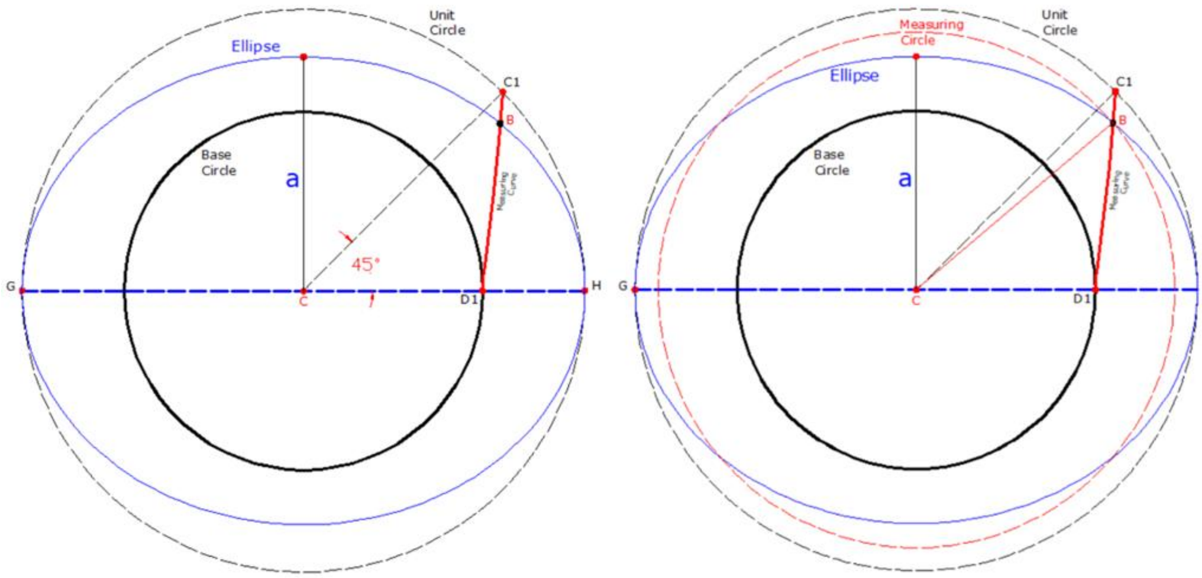

Figure A3. Applying the EPMC with an example of an ellipse, with value of (a = 0.35552655b), and the Measuring Circle's radius is (CB = 0.71663728b).

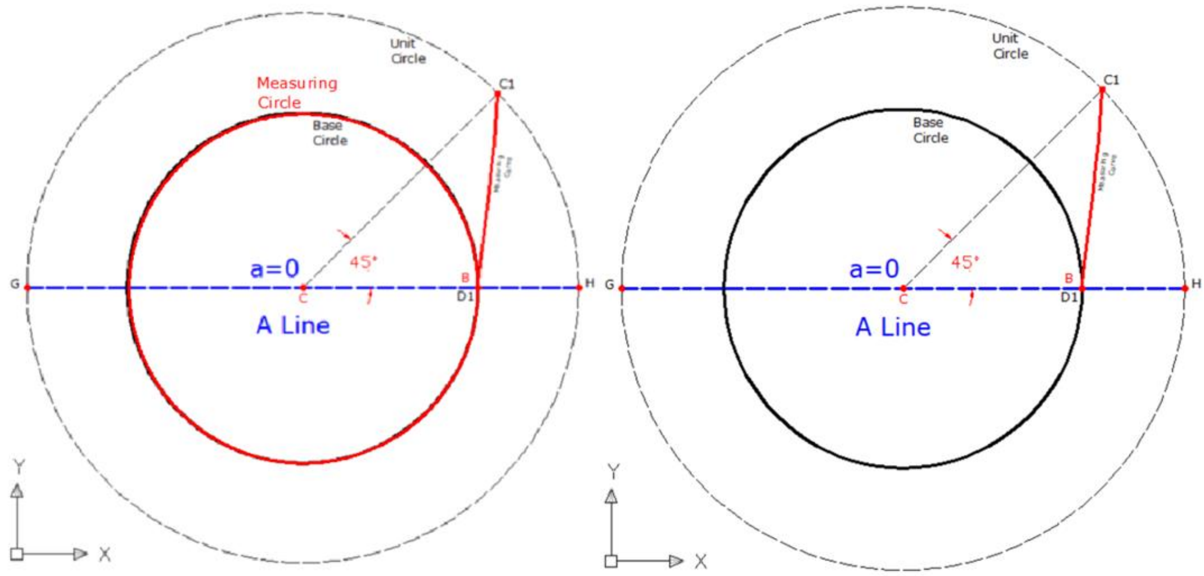

Figure A4. Applying the EPMC with an example of an ellipse, with a value of (a = 0), and the Measuring Circle's radius is (CB = b).

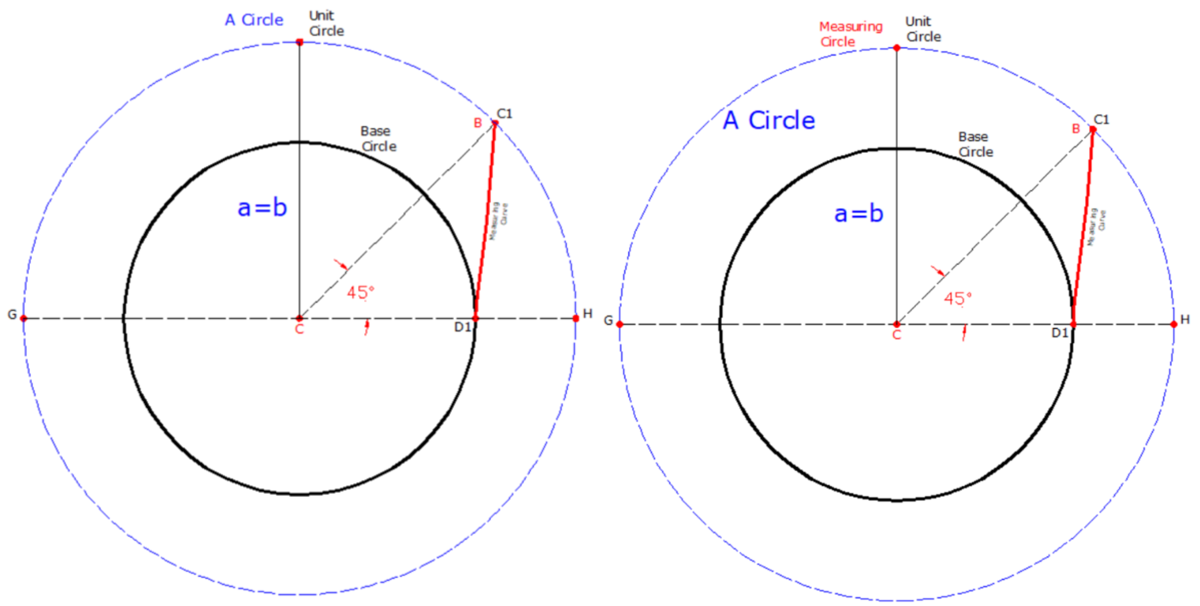

Figure A5. Applying the EPMC with an example of an ellipse, with a value of (a = b), and the Measuring Circle's radius is $\left(CB = \frac{2b}{\pi}\right)$.