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Bridging readiness and reasoning: A mixed methods study on learning readiness and students' argumentation in mathematics

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Abstract:
This study aimed to investigate the relationship between learning readiness and students' argumentation patterns in mathematics learning, and to describe these patterns using Toulmin's argumentation model. Using a mixed-methods approach with a parallel convergent design, this study was conducted in class XI of SMA Negeri 1 Gresik in 5 parallel courses. Subjects were selected through purposive sampling, resulting in three students with different learning readiness levels: advanced, developing, and needing guidance. Data were obtained through tests, think-aloud, and interviews. The data obtained consisted of quantitative and qualitative data. Quantitative data in the form of student argumentation ability scores were subjected to statistical tests to determine their correlation with the level of learning readiness. At the same time, the qualitative data obtained were identified based on Toulmin's argumentation pattern. Data were analyzed using the Miles and Huberman model, which includes data reduction, data presentation, and conclusion. The results showed a positive correlation between learning readiness and students' argumentation ability in mathematics learning. Additionally, it was found that students with different learning readiness categories exhibited distinct mathematical argumentation patterns. In students with learning readiness who need guidance, the argumentation patterns used are claim, ground, and warrant. Students with developing and advanced learning readiness exhibit argumentation patterns, which include claims, bases, warrants, support, and qualifications. This study differs from previous studies in that learning readiness is measured by the ability to understand prerequisite material, rather than general learning readiness.

Keywords: Mathematical Argumentation; Learning Readiness; Toulmin Argumentation Pattern.

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Introduction

In the 21st century, learning focuses not only on knowledge but also on skills that play a crucial role in various aspects of life. Purba et al. (2022) state that the main skills in 21st-century learning are the 4Cs: critical thinking, creativity, communication, and collaboration. These skills are necessary to face challenges and capitalize on opportunities in the era of rapidly developing advances in information and communication technology (Amran et al., 2019). Therefore, teachers are expected to apply the 4C approach in the learning process (Fauziyah et al., 2021). One of the main skills that needs to be developed is critical thinking, which plays a crucial role in problem-solving. Students' problem-solving abilities can be observed in how they solve problems (Fauziyah et al., 2025), and this skill needs to be trained so that students can apply it in various aspects of life (Nasution & Afrianti, 2022).

According to Rafiek, Noortyani, and Abbas (2022), critical thinking involves reviewing ideas, selecting the most effective steps, and evaluating and determining the most appropriate solution. Students who possess critical thinking skills can help them understand and analyze complex problems or objects, enabling them to make informed decisions and draw conclusions (Aslamiah et al., 2021; Lubna et al., 2023; Yu & Zin, 2023). According to Isti, Agoestanto, and Kurniasih (2017), critical thinking skills can help students enhance their understanding of mathematics. It is a solution to help students facing the challenges of mathematical problems (Fauziyah et al., 2019). According to Hasibuan and Surya (2016), the ability to think critically is the foundation for analyzing arguments and developing logical thinking patterns. Therefore, familiarizing students with critical thinking is an effective way to enhance their understanding of mathematical material and train them in the argumentation process, enabling them to solve problems based on existing data.

In mathematics, students need arguments to understand concepts, explain them logically, and determine the best way to solve problems. Argumentation plays a crucial role in mathematics (Reuter, 2023). Argumentation ability encompasses students' skills in constructing arguments, expressing agreement or disagreement with an issue, identifying problems, selecting solutions, and evaluating arguments (Indrawatiningsih, 2018). According to Indrawatiningsih et al. (2020), argumentation ability is a thinking skill that is useful for students, especially in mathematics. This subject requires students to convey statements, both verbal and written, supported by relevant data and Theory to understand mathematical concepts correctly (Indrawatiningsih, 2018; Indrawatiningsih et al., 2019).

The argumentation structure can be identified using the argumentation pattern proposed by Stephen Toulmin, known as the Toulmin Argumentation Pattern (TAP). According to the Toulmin Model in research (Vandoulakis & M Vandoulakis, 2018), there are six interrelated components for argument analysis: Claim, Ground, Warrant, Backing, Rebuttal, and Qualifier. According to Toulmin's Argumentation Pattern, arguments consist of claims, data, and warrants, while the argument components of support, rebuttal, and reasons serve as the criteria for constructing a complete argument. Additionally, arguments are the outcome of the argumentation process (Indrawatiningsih et al., 2019). The Toulmin argumentation pattern is the right choice for analyzing arguments because it is beneficial for measuring someone's argumentation ability. Erduran, Simon, and Osborne demonstrate that TAP is well-suited for researchers to identify arguments and assess argument quality (Anita et al., 2021). One way to improve argumentation skills is to apply specific learning models. According to Pratiwi et al. (2019), the Problem-Based Learning model, combined with argumentation skills, can enhance students' understanding of concepts. The results of this research indicate that argumentation skills in problem-based learning are relatively high, specifically 0.639, which suggests a strong relationship between argumentation skills in problem-based learning.

Apart from implementing specific learning models, other research indicates that differentiated learning has an impact on students' critical thinking abilities, particularly in terms of their mathematical argumentation skills. Differentiated learning has been proven to be more effective in improving students' critical and creative thinking abilities compared to conventional teaching (Lestari et al., 2024). The application of differentiated learning is considered highly effective in enhancing students' understanding of mathematical concepts and is found to be interesting by students (Siregar et al., 2021). According to Islam and Aziza (2024), one of the most relevant models is the differentiated learning model, as it adapts teaching to the needs, interests, and learning styles of each student, allowing them to learn more effectively. According to Ningsih, Sit, and Rakhmawati (2024), students with diverse abilities can achieve their maximum potential in critical and creative thinking through differentiated learning. This differentiated learning approach can be a solution to overcoming diversity, as it involves instruction that enables teachers to achieve learning success in addressing diverse student needs (Pozas, Letzel, & Schneider, 2020).

One of the learning needs of students that needs to be considered to create differentiated learning is readiness to learn (readiness). One aspect of knowing readiness to learn is initial ability. Initial abilities refer to the basic skills that students must possess before beginning new learning, reflecting their readiness to engage with the material presented by the teacher (Mulyono et al., 2018). To measure this ability, a diagnostic assessment is conducted, which includes questions regarding prerequisite material (Gustavo et al., 2019). Teachers need to understand students' learning readiness to provide instruction that meets their individual learning needs.

However, the facts on the ground show something different. Based on the results of observations and interviews with mathematics teachers in Class XI of SMA Negeri 1 Gresik, it is evident that the mathematical argumentation skills of students in Class XI are still not optimal. It is apparent during learning, where students understand the material but struggle to explain it again or articulate arguments that support their answers. Likewise, in the learning process, students are not accustomed to conveying their arguments when solving mathematical problems. According to Syerliana et al. (2018), the low argumentation skills of students are attributed to several factors. First, students are not accustomed to working on problems that require argumentation skills, as they typically work on math problems that only involve number replacement. Second, the learning model applied in the classroom does not provide sufficient support for developing students' argumentation skills. Third, teachers lack clear guidelines on how to assess and develop students' argumentation skills. Based on the study results, Indrawatiningsih et al. (2020), most students fail to solve problems completely due to procedural and conceptual errors in proving valid arguments. The results of another study by Syerliana et al. (2018) showed that students' scientific argumentation skills remained low, as evidenced by an average claim score of 54%, a data score of 38%, a guarantee score of 29%, a support score of 35%, and a rebuttal score of 35%.

Previous research conducted by Dagdag and Calimag (2023) stated that learning readiness has a positive influence on learning outcomes. However, the learning readiness referred to in this study is mental, social, emotional, and physical readiness. Another study conducted by Dangol and Shrestha (2019) found that learning readiness is a fundamental factor in improving students' academic achievement. The readiness measured in this study is related to motivation, physical, and psychological health. Another study, conducted by Tang et al. (2021), found that learning readiness significantly influences the effectiveness of learning. Learning readiness in this study encompasses social, emotional, and environmental factors.

Based on previous research, learning readiness is measured by combining physical and psychological readiness factors. In contrast, this study measures learning readiness through students' understanding of prerequisite material using diagnostic assessments. It is based on the characteristics of mathematics, which are hierarchical, so researchers are very concerned with learning readiness on prerequisite material from the topic to be taught. Additionally, other studies have linked learning readiness more closely with academic ability or learning outcomes. In this study, it is connected to students' argumentation patterns. Because researchers believe that students can convey arguments when studying new material if they understand the basic concepts (prerequisites) needed to grasp the new idea, the novelty of this study lies in its approach, which differs from previous studies. This research aims to determine the correlation between learning readiness and students' argumentation patterns and to describe students' mathematical argumentation patterns based on Toulmin's argumentation patterns in terms of learning readiness. This research aims to increase insight and contribute thoughts on the relationship between students' learning readiness and their argumentation abilities, as well as the differences in argumentation patterns across each category of learning readiness. Apart from that, it can be used as input for teachers to improve students' argumentation skills in mathematics learning and as a reference for subsequent research.

Research Methods

This study employs a mixed-methods approach with a convergent parallel design. According to Creswell, mixed-methods research is a type of research that combines qualitative and quantitative methods for data collection and analysis within a single study. This type of research allows researchers to understand complex phenomena qualitatively and explain these phenomena through numbers, diagrams, and fundamental statistical analysis (Creswell, 1999). Meanwhile, convergent parallel design is a research design in which researchers collect quantitative and qualitative data in parallel or together. This research was conducted at SMA Negeri 1 Gresik in November 2024. For quantitative data, learning readiness and argumentation skills were assessed using data from 150 grade XI students (5 classes) of SMA Negeri 1 Gresik. Samples were taken randomly from 12 existing courses. Participants were given an initial assessment, also known as a cognitive diagnostic assessment, related to prerequisite material to map groups based on learning readiness. The results of the first assessment were processed and categorized into three groups, as in Table 1.

Table 1. Initial Assessment of Learning Readiness Score Categories

No.	Readiness to Learn	Assessment Score Interval
1	Need guidance	$x \leq 60$
2	Develop	$61 \leq x < 90$
3	Advanced	$x \geq 90$

Based on Table 1 above, participants were grouped into three groups based on the results of the initial assessment, namely students with the learning readiness category need guidance (skor $x \leq 60$), the developing learning readiness category (skor $61 \leq x < 90$), and the advanced learning readiness category (skor $x \geq 90$) with a score interval of 0 - 100. To determine the subject of qualitative research, the researcher selected three students from one class with different levels of learning readiness. Specifically, one student was chosen from each learning readiness group using the purposive sampling technique. The purposive sampling technique is a sampling method determined by the researcher, taking into account specific considerations relevant to the research's needs.

The research data were collected using several techniques. To obtain quantitative data related to learning readiness and argumentation skills using written tests. Meanwhile, quantitative data collection related to students' argumentation patterns was carried out, using both argumentation ability test results and interview data. This interview was conducted to dig deeper into the argumentation indicators used in this study. In addition, interviews were conducted as supplementary data to complement the existing data. As a reference in conducting interviews, interview guidelines were prepared to confirm students' argumentation skills based on the results of written tests. Interview data was transcribed for further analysis to answer the formulation of the problem in this study. The indicators used to identify students' argumentation patterns in this study were Toulmin's argumentation indicators, modified from the research indicators presented by Vandoulakis and Vandoulakis (2018), as shown in Table 2 below.

Table 2. Mathematical Argumentation Component Indicators

Argumentation Components	Indicator	Explanation
Claim	Claims conclusions, namely, statements that need to be established.	Students write or convey their assumptions related to the given questions.
Ground	Basis, facts, evidence, and data, namely facts referred to as the basis for a claim.	Students write or convey assumptions that are built based on existing data and facts.
Warrant	Reasoning that enables one to move from facts or data to assertions.	Students write or convey assumptions accompanied by data and connect the data with claims.
Backing	Additional reasoning aimed at confirming the statements expressed on the basis. It is necessary when the warrant itself is not convincing enough.	Students write or submit supporting evidence stating that the warrant is valid.
Qualifier	Words and statements that express the author's level of confidence in their statement.	Students write or convey reasons as justification for statements accompanied by evidence.
Rebuttal	Reasoning that shows the possibility of conditional truth or incorrectness of a proposition from a basis.	Students write or submit statements that can be in the form of rebuttals or exceptions to claims.

To ensure data validity, the collected data were then compared using technical triangulation. In this study, technical triangulation was employed by comparing data obtained from written tests, think-aloud protocols, and interviews. Data from written test results were verified with think-aloud data and reconfirmed through interview data. According to Sciberras and Dingli (2023), validity is determined when the findings from each technique are the same. Technical triangulation is a strategy in qualitative research that aims to increase the validity and reliability of data. It is done by collecting data using more than one method to see if the results obtained are consistent or support each other. In this context, researchers employ three techniques: written tests, which are used to assess participants' ability or understanding in writing, and formal assessments. Think aloud: A method in which participants express what they think when completing tasks or answering questions so that researchers can understand their thinking process. Interview: Used to deepen understanding of the reasons or background of participants' answers, as well as to confirm findings from previous methods.

Data from the written test is verified with data from the think-aloud. It means that researchers not only look at the final answer but also examine the underlying way of thinking, considering whether there is consistency between the written results and the participants' thought processes. After that, the results of the two techniques were reconfirmed through interviews. It is a way to ensure that the data obtained truly reflects the participants' understanding, rather than a coincidence or a one-sided interpretation by the researcher. According to the source, data is considered valid if the findings from each technique show consistent or aligned results. The quantitative data analysis technique employed in this study utilized correlational analysis, specifically Pearson correlation or product-moment tests. Qualitative data were analyzed using the Miles and Huberman model, which involves data reduction, data presentation, and conclusion.

Results and Discussions

This study aims to investigate the relationship between learning readiness and students' argumentation patterns, and to describe students' arguments in terms of Toulmin's argumentation pattern based on their level of learning readiness. To obtain qualitative data, one selected class was categorized by learning readiness level. The results of the categorization of learning readiness levels are presented in Table 3 below.

Table 3. Initial Assessment Result Data

No.	Readiness to Learn	Many Students	Percentage (%)	Research Subject
1.	Need Guidance	7	23.33	S1
2.	Develop	14	46.67	S2
3.	advanced	9	30	S3

Based on Table 3 above, there are three learning readiness groups from a total of 30 students in the same class. The details are as follows: 7 students have learning readiness that requires guidance (23.33%), 14 students have developing learning readiness (46.67%), and nine students have advanced learning readiness (30%). Based on the results of the grouping, one student was selected from each category of learning readiness using a purposive sampling technique to serve as the research subject. To determine their argumentation ability, students were given a test presented in the LKPD. Furthermore, the selected research subjects were identified as related to their argumentation patterns during mathematics learning, especially in group discussion activities when working on Student Worksheets (LKPD). The argumentation ability of the research sample will be subjected to statistical analysis using a Pearson test to determine its correlation with learning readiness. Meanwhile, the argumentation ability of the research subjects will be analyzed based on the Toulmin argumentation pattern indicator (Toulmin's Argumentation Pattern).

The following is a correlation analysis examining the relationship between learning readiness and students' argumentation patterns. Before conducting the correlation test, a normality test was performed as a prerequisite. The results of the data normality test are presented in Table 4 below.

Table 4. Data Normality Test Results

Test of Normality	**Kolmogorov-Smirnov[a]**			**Shapiro-Wilk**		
	Statistic	**df**	**Sig.**	**Statistic**	**df**	**Sig.**
Learning Readiness	0.053	150	0.200*	0.982	150	0.057
Argumentation Sklis	0.179	150	0.075	0.883	150	0.065

Based on the normality tests conducted on the "Learning Readiness" and "Argumentation Skills" data using two methods, namely the Kolmogorov-Smirnov and Shapiro-Wilk tests, it can be concluded that both variables are assumed to have a normal distribution. This conclusion is drawn from the obtained significance values (p-values) from both types of tests for both variables, where all significance values (0.200 for Kolmogorov-Smirnov on "Learning Readiness", 0.057 for Shapiro-Wilk on "Learning Readiness", 0.075 for Kolmogorov-Smirnov on "Argumentation Skills", and 0.065 for Shapiro-Wilk on "Argumentation Skills") are greater than the commonly used significance level of 0.05. Consequently, the assumption of data normality is met for both tested variables. The implication of these findings is that parametric statistical tests can be employed for further analysis of the "Learning Readiness" and "Argumentation Skills" data. To determine the correlation between the two datasets, a Pearson correlation test was performed. The results of the data correlation test are presented in Table 5 below.

Table 5. Data Correlation Test Results

		Learning Readiness	**Argumentation Skills**
Learning Readiness	Pearson Correlation	1	0.963**
	Sig. (2-tailed)		0.000
	N	150	150
Argumentation Skills	Pearson Correlation	0.963**	1
	Sig. (2-tailed)	0.00	
	N	150	150

Based on the Pearson correlation analysis presented in the table, a significant and strong relationship is revealed between the two measured psychological constructs: "Learning Readiness" and "Argumentation Skills." The recorded Pearson correlation coefficient of .963 indicates a very high degree of linear association between these two variables. This value approaches 1, which represents a perfect positive correlation. The interpretation of this correlation coefficient magnitude suggests that there is a strong tendency for individuals with a high level of learning readiness also to demonstrate a high level of argumentation skills, and vice versa. In other words, these two aspects appear to be closely intertwined and substantially interconnected within the population represented by this sample.

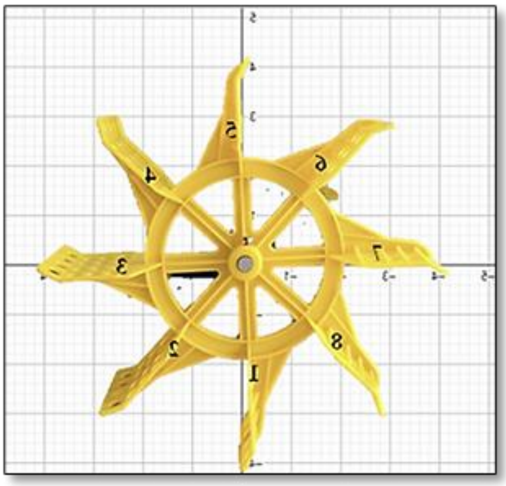
Furthermore, the statistical significance of this correlation is confirmed by the very small p-value (Sig. 2-tailed) of .000. This p-value is far below the conventional significance levels (e.g., 0.05 or even 0.01), providing strong evidence to reject the null hypothesis that there is no linear correlation between "Learning Readiness" and "Argumentation Skills" in the population. In other words, the probability of obtaining a correlation of .963 by chance if there were no correlation in the population is minuscule (less than 0.001). Therefore, it can be concluded with a high degree of confidence that the strong relationship between these two variables is not merely an artifact of sampling but likely reflects a real pattern that applies to the broader population from which this sample of 150 participants was drawn.

Overall, the results of this Pearson correlation test provide strong evidence of a positive and significant relationship between "Learning Readiness" and "Argumentation Skills" within the studied sample group. The very high strength of the relationship underscores the importance of considering both of these aspects simultaneously in the context of education and the development of intellectual skills. The results of the qualitative mathematical argumentation analysis of the research subjects are as follows.

1. *Argument Subject 1 (S_1)*

To answer the question in Activity 1, S_1 observed the image in the student worksheet (LKPD). Then, S1 told their group members that the image represents a type of geometric transformation called rotation. Below is the transcript of the conversation that took place during the discussion.

S_1 : Oh, this is a picture of a windmill. Its movement is to the left, opposite to the direction of the clock hands.
P : What type of geometric transformation is this?
S_1 : Rotation
P : Why is it called a rotation?
S_1 : Because there is a turning movement.
P : What guarantees that the windmill’s motion is a rotation? Are there any other reasons?
S_1 : Because there is a turning from one point to another, it is called a rotation.
P : Is there any other reason why this event is called a rotation?
S_1 : Hmm.



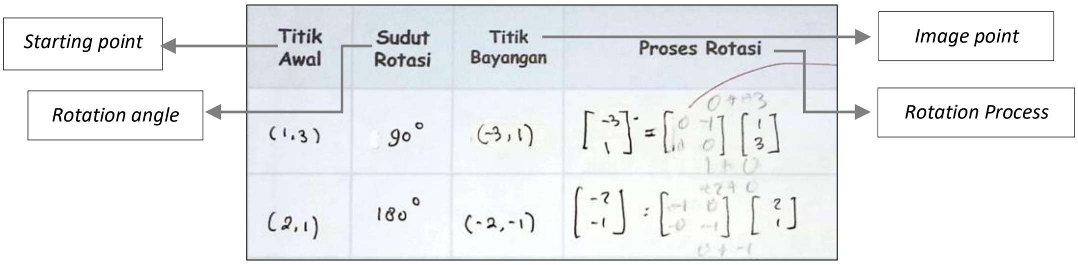
Picture 1. The windmill in Activity 1

In the transcript of the conversation above, a pattern of argumentation from S1 is evident. S1 has claimed that the windmill event is a type of geometric transformation, specifically a rotation. S1 also presented data or facts related to the windmill event, namely the presence of a rotating point that moves as the windmill spins. S1 further strengthened the claim by linking it to the revealed fact that if there is a rotation from one point to another, the event is referred to as a rotation. From this, it can be concluded that in Activity 1, S1 has reached three components of an argumentation pattern: claim, ground, and warrant. However, S1 did not get the backing component, as when asked for another reason to support the claim, S1 was unable to respond. The qualifier component also did not appear, as S1 could not explain any qualifications or limitations of the claim. Likewise, the rebuttal component was not present because S1 was unable to explain what would happen if the claim were challenged.

Next, in the discussion for Activity 2, S1 and their group members read and attempted to understand the given problem. In the task, students were asked to determine an arbitrary starting point, define the rotation angle, find the image point, and describe the rotation process. Then, they had a discussion, and the researcher conducted a Q&A session with S1 as follows:

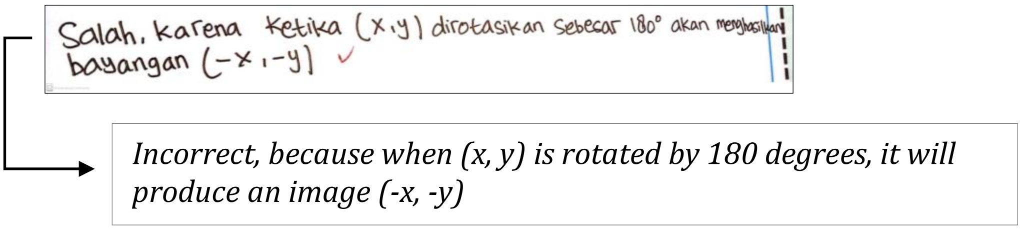
P : Has the initial point been determined?
S_1 : Yes, Ma'am.
P : Which one?
S_1 : This one, Ma'am (while pointing to the completed worksheet).
P : Then, what were you asked to do with that point?
S_1 : The point (x, y) is rotated with the center of rotation at (0, 0) by 180°.
P : What is the image of that point after rotation?
S_1 : (Looking at the previous page of the worksheet) "Oh... this one has a 180° angle. If it starts at (2, 1), it becomes (-2, -1), which means if it's (x, y), it becomes $(-x, -y)$."

S1's answer on the worksheet can be seen in the following Picture 2.



Picture 2. Activity 2 in LKPD from S1

From the think-aloud excerpt and Picture 2, it is evident that S1 can make a claim based on the data. In this case, the claim in question is the answer to the question given. S1 claims that point (*x*, *y*) rotated with the center of rotation (0, 0) by 180° produces a shadow point (-*y*, *x*), which is a false statement. S1 also provides reasons or explanations that support the claim. S1 achieves this by connecting the claim he provided with previously obtained data, where the previously received data states that if point (x, y) is rotated with the center of rotation at (0, 0) by 180°, it produces a shadow point at (-*x*, -*y*). The relationship between claim and ground in the Toulmin argumentation pattern includes the warrant component. The warrant that guarantees the claim must be logical and relevant to the claim submitted, where this warrant can be a definition or theorem (Arifin et al., 2023). This warrant component is conveyed explicitly by S1 in a written answer. Likewise, the claim component is expressed in writing in the LKPD answer. The claim and warrant components are illustrated in Picture 3 below.



Picture 3. Answer from S1

In addition to the answers explained above, S1 did not provide any other supporting data to confirm whether the claim he gave was accurate. S1 also stated that he was not yet sure about the claim he gave. Based on the Toulmin argumentation pattern, the argument presented by S1 lacked supporting components because he had not provided evidence to support the truth of the warrant. Meanwhile, the qualification component did not appear in S1's argument because he was not yet sure about the claim he made. It can be seen based on the results of the researcher's interview with S1. The following is an excerpt:

P : *Can you show evidence that if (x,y) is rotated by 180°, it will produce an image (-x, -y)?*

S_1 : *How about that, Ma'am? I just looked at this (pointing to the LKPD, as shown in Picture 1) and made an educated guess.*

P : *Are you sure about the answer you wrote?*

S_1 : *Hmmm... I'm still not sure, Ma'am. Perhaps I'm a bit uncertain that the statement in the question is incorrect, but I'm confused about how to explain it.*

Based on the description above, it is evident that the S1 argumentation pattern consists only of a claim, ground, and warrant, without any backing, qualifier, or rebuttal components. It aligns with the opinion of Utomo et al. (2019), which states that the quality of students' argumentation tends to be dominated by basic argumentation skills, consisting of a claim, a warrant, and data or grounds.

2. Argument Subject 2 (S2)

To answer the question in Activity 1, S2 observed the image in the student worksheet (LKPD). Then, they discussed it with their group members and wrote down their answers on the provided worksheet. After that, the researcher interviewed S2. Below is the transcript of S2's conversation during the discussion:

P : What image is shown in the worksheet for Activity 1?
S_2 : A windmill.
P : What type of geometric transformation is occurring?
S_2 : Rotation.
P : Why is it called a rotation?
S_2 : Because there is a spinning motion.
P : What confirms that the windmill's movement is a rotation? Are there any other reasons?
S_2 : In a windmill, the blades spin around a central axis or shaft.
P : Is there another reason why this event is considered a rotation?
S_2 : The parts of the blades rotate in a circular path around that center.
P : What if the windmill changes shape while spinning due to wind pressure? Can it still be categorized as a geometric rotation that preserves shape?
S_2 : Umm... I don't know, ma'am.

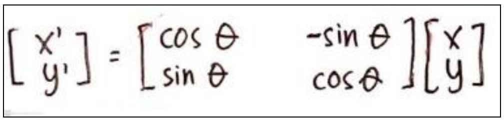
From the transcript above, it is evident that S2 has demonstrated several components of an argumentation pattern: claim, ground, warrant, backing, and qualifier. S2 claimed that the windmill event is a type of geometric transformation, specifically, a rotation. S2 identified a fact or data point, which is that the windmill involves rotation. S2 also linked this fact to strengthen the claim (warrant). S2 reached the backing stage of argumentation by providing additional supporting data for the claim, namely, that the windmill blades spin around a central axis.

Furthermore, S2 reached the qualifier stage by setting a condition or limitation on the claim, stating that the blades rotate in a circular path around the center. However, S2 was unable to respond to a counter-question or challenge, indicating that S2 had not yet reached the rebuttal stage of argumentation. In the second worksheet (LKPD) activity, the same pattern was observed during the in-depth interview; S2 had already reached the same argumentation structure.

P : Has the initial point been determined?
S_2 : Yes, ma'am.

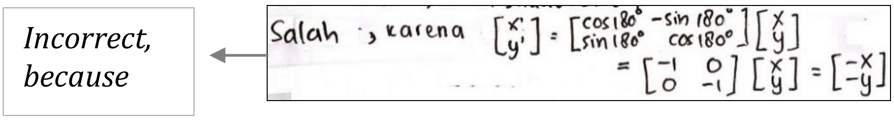
P : Which one?
S_2 : This one, ma'am (while pointing to the completed worksheet).
P : What is done with that point next?
S_2 : The point (x, y) is rotated with the center of rotation at (0, 0) and a rotation angle of 180°. Is the image point really $(-y, x)$? (Looking at the rotation concept in the worksheet) We already found this formula earlier, so we need to substitute θ with 180° to see the image point.
P : How is the process? Is it correct that the point (x, y) becomes $(-y, x)$? What's the reason?
S_2 : A general rotation matrix for an angle θ about the point (0, 0) can be used by multiplying matrices.
P : What does that look like?
S_2 : (writes down the rotation concept as shown in Picture 4)
P : Then, if the point (x, y) is rotated with center (0, 0) and angle 180°, is the image point really $(-y, x)$?
S_2 : Hold on, let me multiply the matrices first. (produces Picture 4)
P : Thus, is the image really $(-y, x)$?
S_2 : No, that's incorrect. The correct image is $(-x, -y)$.

The concept of rotation in LKPD, which is used as supporting data by S2, is illustrated in Picture 4 below.



Picture 4. Rotation Concept in LKPD S2

From the think-aloud excerpt and Picture 4, it can be seen that S2's argument already contains basic components. Based on the data, S2 was able to claim the form of an answer to the given question, even though the initial claim was incorrect. After obtaining supporting data using the concept illustrated in Picture 5, S2 eventually made a revised claim, stating that the statement in the question was wrong. Furthermore, S2 became confident that the final claim was accurate after completing the matrix multiplication shown in Picture 5.



Picture 5. Answer from S2

To further understand the meaning behind S2's answer, the researcher conducted an interview with S2. Below is an excerpt from the interview:

P : Please explain the meaning of the answer you wrote. How did you conclude that the statement was incorrect?

S_2 : Well, Ma'am, earlier, we learned the formula for finding the image of a point. Thus, I plugged the information from the question into that formula. The resulting image point is $(-x, -y)$. In the question, the image point is given as $(-y, x)$, but according to the calculation, it should be $(-x, -y)$. So the answer is incorrect, Ma'am.

P : Are you very sure about that?

S_2 : Yes, ma'am. Because the cosine of 180 degrees is -1, and the sine of 180 degrees is 0. Then, the matrix multiplication is also correct. So I'm very sure.

From the interview excerpt above, it is clear that S2 was able to provide reasoning that supported the claim they made. S2 was able to connect the claim with the ground or supporting data previously obtained, namely, by using the rotation formula to find the image point. The connection between the claim and the ground, in Toulmin's model of argumentation, is referred to as the warrant component. Furthermore, S2 also reached the backing stage by demonstrating that the calculations were accurate, including the correct values of cos 180°, sin 180°, and proper matrix multiplication, which resulted in the correct answer. The qualifier component was also present, as S2 expressed confidence in the answer. It can be seen from the following interview excerpt:

P : What are the conditions or limitations for using that formula?

S_2 : It can be used as long as the center of rotation is at the origin (0, 0).

P : What if the center of rotation is not at the origin (0, 0)? What would the solution look like?

S_2 : I don't know, ma'am.

The last question indicates that S2's argument did not reach the **rebuttal** stage. However, this is already a substantial achievement for a student at this stage of development. It aligns with Lin's view that not all arguments in mathematical proofs require a rebuttal (Lin, 2018).

3. Argument Subject 3 (S3)

In Activity 1, Subject 3 observed the image of a windmill in the worksheet (LKPD). As the group leader, they tried to engage their group members in discussion and answer several questions on the provided worksheet. The group's discussion results were written down on the LKPD sheet. Afterward, the researcher interviewed with S3. Below is the transcript of S3's conversation during the discussion:

P : Have you finished the discussion?

S_3 : Yes, ma'am.

P : What image is shown in the worksheet for Activity 1?

S_3 : A windmill.

P : What type of geometric transformation is occurring?

S_3 : Rotation

P : Why is it called a rotation?

S_3 : Because there is a spinning movement.

P : Can you give an example?

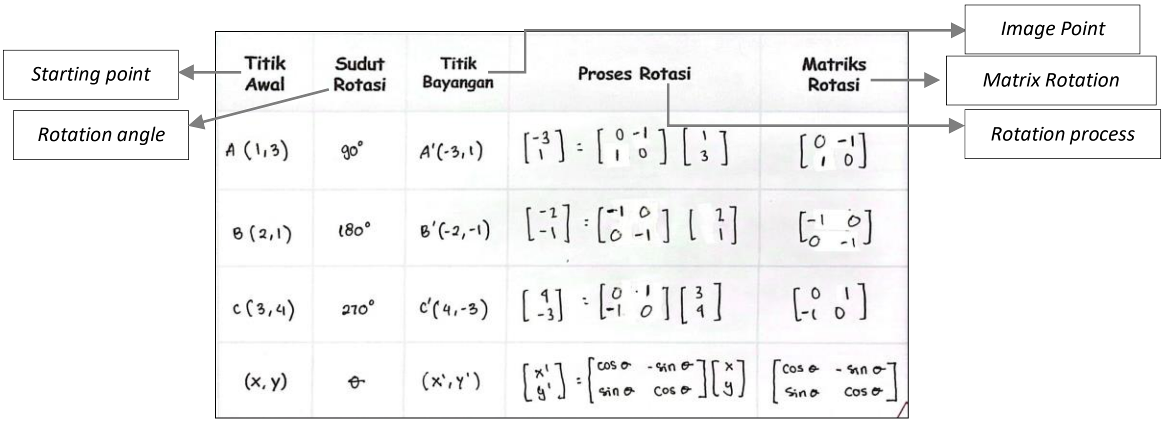
S_3 : For example, the number 1 can rotate and be in a different position.
P : What guarantees that the windmill's movement is a rotation? Are there other reasons
S_3 : In a windmill, the blades rotate around the central axis, also known as the coordinate axis.
P : Are there other reasons why this event is referred to as a rotation?
S_3 : It rotates in a circular path around the center of the coordinate axis.
P : Can the windmill still be called a rotation if the center of rotation is not fixed or shifts due to unstable wind?
S_3 : As long as there is rotation, Ma'am.

From the transcript above, it appears that S3 has reached the argumentation pattern stages of claim, ground, warrant, backing, and qualifier. S3 claims that the windmill event is a type of geometric transformation, specifically rotation. S3 finds the data or fact that the windmill involves rotational movement. S3 also connects this fact to strengthen the claim (warrant). S3 reaches the backing stage by mentioning additional supporting data related to the claim that the event is a rotation because the windmill blades rotate around the central axis or shaft.

Furthermore, S3 reaches the qualifier stage by setting a limitation or qualification on the claim, stating that the blades rotate in a circular path around the center. However, S3 provides a less accurate answer at the rebuttal question stage, saying that the windmill can still be called a rotation if the center of rotation is not fixed or shifts, as long as there is rotation. Up to this point, S3 has not reached the rebuttal stage of argumentation. Next, in Activity 2, S3 gathered supporting data to produce a conclusive answer. The supporting data was obtained by reading the information in the problem and observing previous activities presented in the worksheet. It was evident from the think-aloud shared by S3. Below is an excerpt from S3's think-aloud recording:

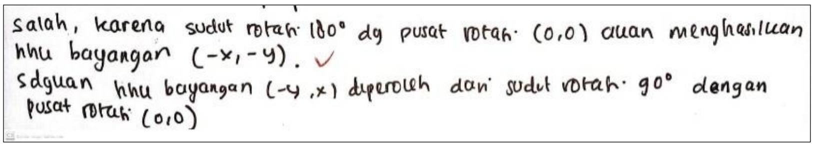
P : Have you determined the initial point?
S_2 : Yes, ma'am.
P : Which one?
S_2 : This one, ma'am (while pointing to the completed worksheet).
P : What do you do with that point next?
S_2 : Determine the rotation angle, Ma'am.
P : How many degrees?
S_2 : 90, 180, and 270 degrees.
P : The point (x, y) is rotated around the center at (0, 0) by an angle of 180°. Is the image point $(-y, x)$ correct or incorrect?
S_2 : If the angle is 180°, the initial point (2, 1) becomes (−2, −1); therefore, if (x, y), then the image is $(-x, -y)$.
P : If the rotation angle is 90 degrees, what is the image?
S_2 : If the angle is 90°, the initial point (1, 3) becomes (−3, 1), so if (x, y), the image is $(-y, x)$.
P : So, for the point (x, y) rotated 180 degrees, is the image point $(-y, x)$ correct or incorrect?
S_2 : It is incorrect because if the rotation angle is 180°, the image point of (x, y) is $(-x, -y)$.
P : If the image point is $(-y, x)$, what is the rotation angle?

S_2 : The rotation angle is 90 degrees.



Picture 6. Activity 2 in LKPD from S3

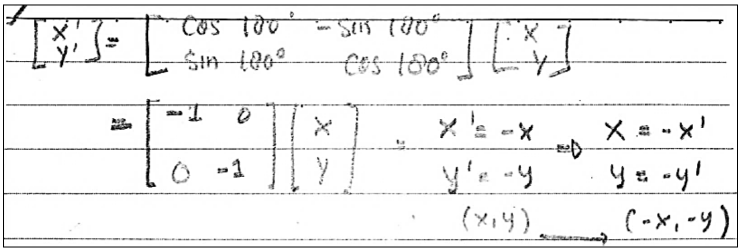
From the excerpts of the think-aloud and Picture 5, it can be seen that S3's argument already contains the basic components. Based on this data, S3 can make a claim, which is the answer to the given question. The statement in the question states that the point (x, y) is rotated about the center of rotation (0, 0) by 180°, resulting in the image point $(-y, x)$. S3 makes a claim stating that the statement given in the question is incorrect. In addition to making the claim, S3 also provides reasons or explanations to strengthen the claim. S3 achieves this by connecting the claim with the previously obtained data. The answer that supports the claim is that if the point (x, y) is rotated about the center (0, 0) by 180°, the resulting image point is $(-x, -y)$, whereas the image point $(-y, x)$ results from a 90° rotation around the center (0, 0). It shows that in S3's argument, there is a warrant component, where the warrant represents the connection between the claim and the data obtained by expressing the relationship between the known information and the data in a statement. S3 explicitly conveys this warrant component in the written answer. Similarly, the claim component is also expressed in writing in the LKPD answer. The claim and warrant components are illustrated in Picture 7 below.



Incorrect, because a 180-degree rotation with the center of rotation at (0, 0) will produce an image (-x, -y). Meanwhile, the image (-y, x) is obtained from a 90-degree rotation with the center of rotation at (0, 0).

Picture 7. Answer from S3

In addition to the answer presented above, S3 also provides other supporting data that proves whether the warrant he gave is correct. S3 wrote the calculation to find the shadow point using the rotation formula previously found. It is evident in Picture 8 below.



Picture 8. Additional Answers from S3

From Picture 8 above, it can be seen that S3 can provide backing or additional evidence supporting that the warrant is correct. Furthermore, in S3's argument, the qualifier component has appeared, indicated by the limitations on the conditions under which the claim is true, as specified by S3. It can be seen from the researcher's interview with S3. The excerpt from the interview is as follows:

P : Is the result of a 180-degree rotation always $(-x, -y)$ if the center of rotation is not at the point (0, 0)?

S_2 : I don't think so, ma'am. The result of a 180° rotation changes if the center of rotation is not at (0, 0).

P : How so?

S_2 : I don't know, ma'am.

The last question shows that S3 has not yet reached the rebuttal stage of argumentation. When the researcher asked what happens if the center of rotation is not at point (0, 0), S3 had not yet considered this stage (Lin, 2018). From the explanation above, it can be seen that the argumentation of the third subject has a different pattern. Generally, the differences in argumentation patterns are illustrated in Table 6 below.

Table 6. Differences in Components of Research Subject Argumentation Based on Toulmin's Argumentation Pattern

Argumentation Components	S1	S2	S3
Claim	✓	✓	✓
Ground	✓	✓	✓
Warrant	✓	✓	✓
Backing	×	✓	✓
Qualifier	×	✓	✓
Rebuttal	×	×	×

S1 demonstrates an argumentation pattern consisting of a claim, data (ground), and reasoning (warrant). S1 claims that the windmill phenomenon represents a geometric transformation, specifically a rotation, supported by the fact that points are turning as the windmill spins. Additionally, S1 explains that a point (x, y) rotated 180° around the origin (0, 0) results in the image point (-x, -y), using this as a basis to strengthen the claim. However, S1 is unable to present other components of argumentation, such as backing (supporting evidence), qualifier (limitations of the claim), and rebuttal (responses to possible counter-arguments). S1 admits to being uncertain about the claim and is unable to provide further justification when asked. Overall, S1's argumentation pattern includes only the basic elements of claim, ground, and warrant, which reflects a common tendency among students to construct arguments limited to the basic structure.

S2 demonstrated a fairly complete argumentation pattern based on Toulmin's model, covering five out of six key components: claim, ground (data or facts), warrant (reasoning), backing (additional support), and qualifier (qualification). S2 claimed that the windmill phenomenon represents a geometric transformation in the form of a rotation, supported by the fact that the blades rotate around a central axis. This claim was strengthened by logical reasoning that connected the observed rotation with the concept of geometric rotation. S2 also provided additional data as support, such as the explanation that the windmill blades rotate in a circular path around a center point, and confirmed the accuracy of this reasoning through correct calculations using cos (180°), sin (180°), and matrix multiplication. S2's confidence in the correctness of the answer indicates the presence of the qualifier component in the argument. Although the initial claim made by S2 was incorrect, through further analysis and reflection on supporting data, S2 was able to revise and reinforce the claim. However, S2 did not reach the rebuttal stage, as they were unable to respond to possible objections or counterarguments to their claim. Nonetheless, S2's argumentation skills are considered strong, although they are still in a developmental phase. It aligns with the view that in mathematical proof, not all arguments necessarily require a rebuttal component.

S3 demonstrates a relatively good understanding of constructing mathematical arguments based on Toulmin's model, covering five of the six main components: claim, ground, warrant, backing, and qualifier. S3 claims that the event occurring in the windmill is a type of geometric transformation called rotation. This claim is supported by observed data or facts, namely the spinning motion of the windmill blades. To strengthen the claim, S3 explains that the windmill blades rotate around the central axis or pivot, following a circular path around the center, indicating that the warrant component is fulfilled. Furthermore, S3 also provides backing or additional support for the warrant. He mentions that the windmill event is considered a rotation because the motion is around a central point. This explanation shows that S3 not only understands the relationship between the claim and the data but can also provide further supporting reasons that strengthen the logic of his argument.

Additionally, the qualifier component appears in S3's argument when he sets limitations or conditions for the validity of his claim. S3 states that a 180° rotation results in the image point (-x, -y) only if the center of rotation is at the origin (0,0). However, if the center of rotation is not at (0,0), the outcome may differ. It demonstrates that S3 recognizes that mathematical claims often apply under specific conditions and can identify these conditions. However, S3 has not yet succeeded in developing the rebuttal component. When asked challenging or counter-questions, such as how the result changes if the center of rotation shifts, S3 is unable to provide an appropriate explanation or counterargument. He only states that the result would be different, but cannot explain how or why this happens. Therefore, it can be concluded that S3's argumentation pattern is quite complete up to the qualifier component but has not yet reached the rebuttal stage, which demonstrates the ability to defend a claim against objections.

Another study that strengthens the results of this study is a study conducted by Solar et al. (2023), which examines the development of student argumentation through the process of discussion, explanation, and defense of the mathematical solutions they propose. The class discussion process will run smoothly if students have a solid understanding of the prerequisite material. Students will find it easy to learn certain materials if they have mastered the prerequisite material. For example, the material on the system of linear equations in two variables requires mastery of prerequisite material on linear equations in one variable, including an understanding of its components: variables, constants, and coefficients. Another study that strengthens the results of this study is a study conducted by Corneli et al. (2019), in which teachers are encouraged to explore students' argumentation skills during the learning process through discussion. Of course, the debate will run well if students understand the previous prerequisite material (Dawson, 2024; Perkins et al., 2016; Rycroft-Smith, 2024).

Other relevant research suggests that Toulmin's argumentation patterns serve not only as a structural analysis tool but also as a bridge to understanding social and epistemic interactions in the argumentation process (Erduran, 2018). Other studies also suggest that the Toulmin argumentation model is effective in enhancing reasoning mechanisms (Gabriel et al., 2020). Additionally, the Toulmin argumentation model can serve as a practical learning tool to develop critical thinking skills, clinical reasoning, and argumentative communication (Fejer et al., 2022). Meanwhile, reasoning and critical thinking skills have a significant positive relationship with self-directed learning readiness. Self-directed learning readiness predicts essential skills of thinking and self-efficacy by 50.5% (Turan & Koç, 2018; Guerin et al., 2021).

Conclusions and Suggestions

Based on the analysis of three students (S1, S2, and S3) using Toulmin's argumentation model, it was found that the completeness of their arguments varied. S1 was only able to construct a basic argument consisting of a claim, ground, and warrant without including additional backing, qualifiers, or rebuttals. In contrast, S2 and S3 demonstrated more complete argumentation patterns, covering five out of the six Toulmin components: claim, ground, warrant, backing, and qualifier, although both were still unable to develop the rebuttal component. It indicates that students' mathematical argumentation skills are still in the developmental stage. Moreover, these findings are supported by previous studies, which emphasize that the development of students' argumentation skills strongly depends on their understanding of prerequisite concepts and the role of discussion in the learning process. The Toulmin model has also been demonstrated to be effective in enhancing students' critical thinking, reasoning, and argumentative communication skills. Therefore, learning activities that encourage argument exploration through conceptual discussions can significantly improve students' readiness for self-directed learning and their self-efficacy.

The results of this study can be used as a consideration for teachers, suggesting that before learning, especially on new topics or chapters, they should first administer a diagnostic test. The diagnostic test is to determine students' learning readiness for mastering the prerequisite materials. Furthermore, learning readiness mapping is conducted, and treatment can be provided before proceeding to new materials. Treatment can be given in groups based on differences in learning readiness levels through a differentiated learning approach.

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