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Journey to the cosmos: Navigating stellar evolution with differential equations

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***Abstract*:**
Differential equations are a fundamental and versatile mathematical tool that finds widespread application across diverse academic disciplines, from physics and biology to economics and engineering. The primary objectives of this report are to demonstrate the application of differential equations in stellar evolution, construct a mathematical model to demonstrate nuclear reactions in a star, and illustrate energy transport within a star. Triangulation was used to prepare this report, with literature studies being the primary method. This study includes several documents and field data analyzed using qualitative research. Through research and observations, two hypothetical case studies illustrate the indispensable application of differential equations in modeling energy transport and nuclear reactions within stars through which the value of luminosity was calculated in a particular star due to both radiative energy transport and convective energy transport while in another star, the helium abundance in the core was estimated to approach a value of 1.195*1077. These differential equations are not only limited to the growth of a lead but also have broader applications that are essential for understanding the chemical composition of the universe and its prolonged evolution. The report also underscores the enduring importance of differential equations in advancing our understanding of the cosmos and their vital role in space exploration and technological innovations.

Keywords: Astrophysics; Differential Equations; Energy Transport; Hertzsprung-Russell Diagram; Nuclear Reaction; Stellar Evolution.

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Introduction

Differential equations stand as a ubiquitous concept in the realms of mathematics, science, and engineering. They formulate an exceptionally effective framework for modeling and comprehending dynamic processes that undergo continuous changes across time or space. They are very suitable for modeling dynamic phenomena whose behaviours change continuously concerning time, space, or other independent variables. Differential equations articulate the alteration rate in a given quantity as a function of the amount and other relevant variables (Johnson, 2012). These equations constitute indispensable tools for resolving real-world challenges and finding applications across various academic disciplines, from physics and biology to economics and engineering. Research on structural health monitoring using differential equations to detect damage or changes in structural properties through vibrations has been ongoing in recent years under the leadership of Doebling and Kazerouni (2021); differential Equations are versatile and capable of describing the rate of change of a wide variety of quantities, including physical, biological and economic processes.

In recent years, the importance of differential equations has extended to disciplines such as artificial intelligence and machine algorithms (Ernst et al., 2021). Astrophysicist J. Homer Lane derived and studied an equation that models the equilibrium of stellar configurations, which Emden later extended. It is known as the Lane-Emden equation that describes the polytrophic models (Abbas et al., 2019). Furthermore, applying differential equations in modeling nuclear reactions in hydrogen-burning stars provides a new perspective on these celestial bodies. The pp reaction, The $d(p,\gamma)^3He$ radiative capture reaction, the $^3He(^3He,2p)^4He$ reaction, the $^3He(\alpha,\gamma)^7Be$ reaction, the $^3He(p,e^+ \nu e)^4He$ reaction, electron capture by ^7Be, pp, and CNO nuclei, the $^7Be(p,\gamma)^8$ B reaction, the spectrum of ^8B neutrinos and the CNO cycle find profound employment of a variety of differential equations to model stellar nuclear reactions (Adelberger et al., 2011). Differential equations have acquired more importance over the past few years, especially in emerging areas like cosmology and astrophysics. Understanding and modeling the intricate dynamics of the cosmos has long relied on the robust framework of differential equations, making them a cornerstone in astronomy. Just as they explore the dynamic behaviour of several systems in physics and mathematics, these mathematical equations offer profound insights into the ever-expanding universe. The roots of this mathematical foundation can be traced back to luminaries like Sir Isaac Newton and Pierre-Simon Laplace, who laid the groundwork for the genuine role of differential equations in cosmological inquiry.

Differential equations serve as an obligatory platform for scientific exploration and engineering endeavours, offering a means to understand the complex workings of celestial bodies, stellar evolutions, and cosmic phenomena. They enhance our ability to model these complex systems and provide a pathway to comprehend the universe's dynamic nature. The level density is among the most essential statistical nuclear properties. It appears in Fermi's golden rule for transition rates and is an indispensable input for the Hauser-Feshbach theory of compound nucleus reactions. Differential equations are used in empirical models and microscopic methods of nuclear-level densities (Alhassid, 2021). In this realm of mystery, uncertainty, and intricacy, differential equations are harnessed to decipher the workings of the unknown. Notably, the works of Augustson et al. (2012) in seeking to make contact with observations and to provide a self-consistent picture of how differential rotation is achieved in the interiors of main-sequence spectral F-type stars have expanded our understanding of the cosmos. Furthermore, the use of differential equations to analyze a discrete ordinates (S_N) discretization of a filtered radiative transport equation (RTE) and the numeric artefacts created by them, known as "ray-effects" in the contemporary is widespread (Hauck & Heningburg, 2019).

In the work of Hou et al. (2015), radiative transfer problems incorporating the variable refractive index have been dealt with through differential approximations. Similarly, Jermyn et al. (2020) deploy differential equations to deal with the scaling of differential rotation in both slowly and rapidly rotating convection zones using order of magnitude methods. The comparison of the first stellar models on the Hertzsprung–Russell diagram, in which convection is treated according to the new scale-free convection theory (SFC theory) against the classical, calibrated mixing-length (ML) theory, is done by using several partial differential equations (Arnett et al., 2015; Pasetto et al., 2016). The textbook "Stellar Structure and Evolution" (Kippenhahn et al., 2012) details how various differential equations play an essential role in stellar evolution.

The complex life cycles of celestial entities, especially stars, from birth to death, meticulously shape the fabric of the universe. Understanding these astronomical occurrences is an exciting field of study and a crucial undertaking in deciphering the origin of elements, the development of galaxies, and the final fate of stars such as our Sun. The key to comprehending stars is figuring out how two fundamental processes, energy transfer and nuclear reactions, occur within them. The intricate differential equations regulating these processes serve as the foundation for models describing stars' structure and development (Dale, 2015). These formulas, rooted in nuclear physics, thermodynamics, and hydrodynamics, provide a mathematical framework crucial for understanding the delicate balance that maintains a star's stability and propels its evolutionary trajectory. Over the decades, astrophysicists have diligently constructed and refined these differential equations, drawing inspiration from observational data and theoretical advancements (Altshuler, 2021; Hodson & Wong, 2014; Kaltenborn, 2023; Penprase & Penprase, 2017; Roychoudhuri, 2011). This continuous pursuit of knowledge, aptly represented by the triangle principle (observations-theory-interpretation), has led to remarkable breakthroughs in understanding stellar phenomena.

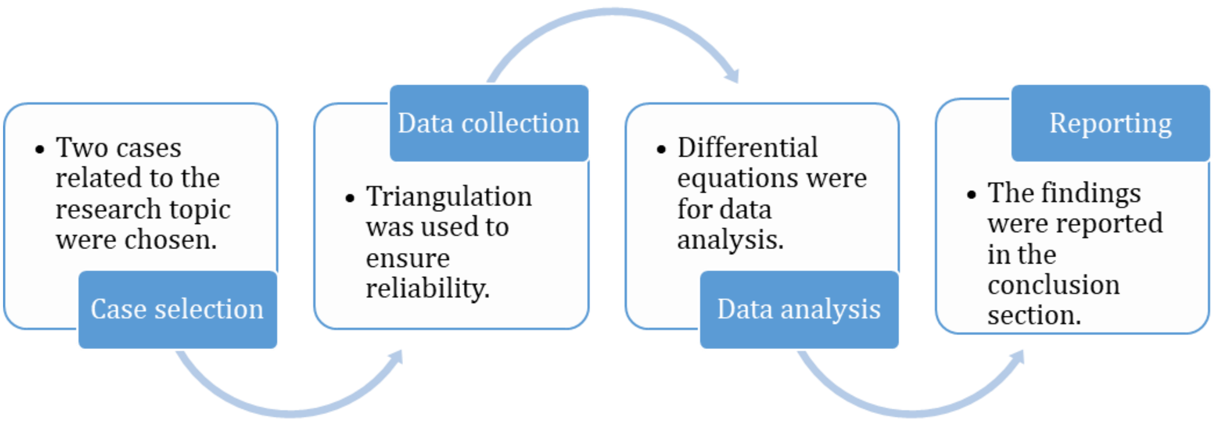
However, despite the immense strides made, significant gaps remain in our theoretical framework. The intricate interplay between radiation, convection, and nuclear reactions, particularly in the late stages of stellar evolution, continues challenging our current models (Clarkson & Herwig, 2021). More precise and detailed observational data is also needed to refine these models. This research aims to address these crucial gaps by developing a novel computational framework that incorporates advanced numerical techniques and the latest theoretical advancements in energy transport and nuclear reactions, conducting a comprehensive analysis of existing observational data to identify discrepancies with current models and refine our understanding of fundamental physical processes and utilizing the newly developed framework to investigate previously unexplored aspects of stellar evolution.

Likewise, this research offers several significant advantages over previous studies. (1) Enhanced accuracy and fidelity: The incorporation of advanced numerical techniques and state-of-the-art physical models will lead to more accurate and reliable predictions of stellar behaviour; (2) Greater predictive power: The novel computational framework will enable us to explore previously inaccessible regions of parameter space, shedding light on unexplored aspects of stellar evolution; and (3) Improved understanding of observational data: The comprehensive analysis of existing observations will provide valuable feedback to refine our models and enhance our understanding of the underlying physical processes.

Research Methods

This paper employs a case study as the primary methodology. A case study is a research method that involves an in-depth examination and analysis of a single case, such as an individual, organization, community, event, or situation. It is a qualitative research approach that aims to provide a detailed and comprehensive understanding of the studied case (Baskarada, 2014; Bogopane, 2013; Cruz & Tantia, 2017; Harrison et al., 2017; Mohajan, 2018). A case study uses triangulation to collect data, which means using several sources, including documents and field data. This method is expected to analyze the collected data using qualitative techniques such as thematic analysis, content analysis, or pattern recognition to derive insights and patterns specific to the case. Some certain steps of the case study involved in writing this paper are as follows.

a. Case selection: Two events related to stellar evolution, namely energy transport and nuclear reactions, were chosen as the cases to be studied in this paper through much contemplation.
b. Data collection: Triangulation was used to gather data for this research paper. The primary modes of data collection were several documents, research papers, and books related to the topic. These sources have been mentioned in the reference section. Several internet archives and e-libraries were also referred to to ensure reliability and validity.
c. Data analysis: Associated differential equations were used to analyze the collected data to identify themes, patterns, and unique aspects of the case, interpret the data, and draw conclusions based on the findings. The equations used were (1) Radiative energy transport equation; (2) Convective energy transport equation; and (3) PP-chain reaction equation. The respective case studies present a further detailed explanation of these equations.
d. Reporting: The case study's findings have been summarized and presented in the conclusion and suggestion section. The descriptions of the case, analysis, and interpretations have been included in it, along with data excerpts.



Picture 1. Research Procedure

In an attempt to give the research a more practical and realistic look, the research results have been presented in the form of a hypothetical case study where the various activities, such as data collection and analyses, are presented in such a way that it seems to have been performed by the hypothetical characters involved in the case studies.

Research Results

Case 1: Understanding Energy Transport in an Evolving Star.

Prelude

Energy transport within stars is a fundamental process that influences their evolution. This case study delves into the mechanisms by which energy is transported within a star as it evolves. We'll follow a team of astrophysicists and scientists as they explore the energy transport equation and determine the star's luminosity, which plays a pivotal role in understanding stellar dynamics.

Case Study Scenario

In 2035, a team of scientists led by Dr. Sarah Leon made significant advancements in telescope technology and data analysis tools. Their mission is to unravel the mysteries of energy transport within stars, particularly as they progress through different stages of their life cycles. For this, the team selects a star called S-T12, a main-sequence star, as their target.

Framework

Radiative energy transport involves the movement of energy through electromagnetic waves, primarily through a star's core and inner layers (Feiden & Chaboyer, 2013; Potekhin, 2014; Steinacker et al., 2013). It occurs via photons constantly emitted, absorbed, and re-emitted as they move through the stellar material. The photons can take a long path as they zigzag out of the star, interacting with particles in the stellar plasma. Convective transport, on the other hand, occurs in the outer layers of stars where temperatures are lower. When the temperature gradient is steep enough, it can drive a convection process. Convection happens when hot, less dense material rises, carrying energy toward the surface, while more fantastic, more dense material sinks back down. This process creates convective cells or "bubbles" within the star, where hot material rises, cools near the surface, and then drops back down to be reheated. In these convective zones, energy is transported more efficiently than through radiation because matter physically moves, carrying the power. The energy transport equation, based on the modes mentioned above of transport, is a crucial component in modeling energy transport within stars. It's defined by a set of differential equations describing the energy flow within a star. These equations consider various parameters, including temperature, density, opacity, etc. The primary equations involved are the radiative energy transport and convective energy transport equations.

Radiative Energy Transport Equation

$L_r = -(4\pi r^2/3k) \cdot dT/dr$

Where:

- L_r is the luminosity due to radiative energy transport.
- r is the radial distance from the centre of the star.
- k is the opacity, representing how effectively the medium scatters and absorbs photons.
- dT/dr is the temperature gradient concerning radius.

Convective Energy Transport Equation:

$L_c = 16\pi/5 \cdot \rho v c_p T \cdot (T/\mu \cdot ds/dr - d\ \ln T/dr)$

Where:

- L_c is the luminosity due to convective energy transport.
- ρ is the density.
- v is the velocity of the convective motion.
- c_p is the specific heat at constant pressure.
- T is temperature.
- μ is the mean molecular weight (Amarsi et al., 2021).
- ds/dr is the specific entropy gradient concerning radius.
- d lnT/dr is the logarithmic temperature gradient concerning radius.

Data Collection

The team collects observational data from various sources, including space-based observatories like the James Webb Space Telescope and ground-based telescopes. These observations include temperature profiles, density measurements, and information on energy generation in the star.

Parameter Estimation

Utilizing the collected data, the team estimates several crucial parameters, such as temperature gradients, density profiles, temperature, radial distance, the opacity of the stellar material, and many others.

The observed parameters are:

- Radial distance (r) = 0.7 $R_\odot$ (where $R_\odot$ is solar radius).
- Opacity (k) = 0.2 cm^2/g
- Temperature gradient (dT/dr) = -10^7k/cm
- Density (ρ) = 0.5 g/cm^3
- Velocity (v) = 1 km/s
- Specific heat at constant pressure (c_p) = 10^7 erg/g/k
- Temperature (T) = 5000K
- Mean molecular weight (μ) = 2

Luminosity Calculation

The star's luminosity is a critical factor in understanding its energy output. The team calculates the luminosity of the S-T12 star using the obtained parameters and energy transport equations.

- *Calculation of Luminosity due to Radiation:*

  $L_r = - (4\pi r^2/3k) \cdot dT/dr$

$L_r = - (4\pi (0.7 R_\odot)^2 / 3 \cdot 0.2 cm^2/g) \cdot (-10^7 K/cm)$
$L_r = - (4\pi \cdot (0.7 \cdot 6.957 * 10^{10} cm)^2 / 0.6 cm^2/g) \cdot (-10^7 K/cm)$
$L_r = - (4\pi \cdot (0.7 \cdot 6.957 * 10^8 m)^2 / 0.06 m^2/kg) \cdot (-10^9 K/m)$
$L_r \approx 4.98 * 10^{28} J/s$
Thus, the value of luminosity (L_r) due to radiative energy transport is approximately $4.98 * 10^{28}$ J/s.

- *Calculation of Luminosity due to convection:*
  $L_c = 16\pi/5 \cdot \rho v c_p T \cdot (T/\mu \cdot ds/dr – d \ln T/dr)$
  For this calculation, the team determines the values of ds/dr (specific entropy gradient) and d lnT/dr (logarithmic temperature gradient) through complex stellar structure models. The obtained values are ds/dr = 0.02 J/g/k/cm and
  d lnT/dr = -0.05 cm^{-1}.
  $L_c = 16\pi/5 \cdot 0.5 g/cm^3 \cdot 1 km/s \cdot 10^7 erg/g/K \cdot (5000 K/2 \cdot 0.02 J/g/K/cm - (- 0.05 cm^{-1}))$
  $L_c = 16\pi/5 \cdot 500 kg/m^3 \cdot 1000 m/s \cdot 1000 J/kg/K \cdot (2500 K \cdot 2000 J/kg/K/m – (-5 m^{-1})$
  $L_c = 2.51 * 10^{16} J/s$
  Thus, the value of luminosity (L_c) due to convective energy transport is approximately $2.51 * 10^{16}$ J/s.

Analysis Results

After conducting extensive calculations and modeling, the team analyzes the results. They explore how different stars' characteristics, such as mass and composition, influence the balance between radiative and convective energy transport through their analysis of the luminosity produced in both modes of transportation. This balance is key to understanding the evolutionary paths of stars.

Cosmological Implications

While this case study focuses on an individual star, the findings have broader cosmological implications. Understanding energy transport within stars is crucial for unravelling the secrets of stellar evolution and, by extension, the development of galaxies and the universe.

Cessation

This case study demonstrates the energy transport equation's importance in deciphering stars' inner workings. By studying how energy is generated, transported, and radiated, scientists can gain valuable insights into celestial objects' life cycles and behaviours. The knowledge gained from this research contributes to our understanding of the universe's past, present, and future.

Case 2: Modeling Nuclear Reactions in a Star

Prelude

Star SA-13 is a hypothetical star with a mass of 5 times that of the Sun and is in the main sequence phase of its evolution. It is composed primarily of hydrogen, and its core temperature and pressure have reached levels where nuclear fusion reactions are occurring. In this case study, we follow a research team that aims to model the nuclear reactions in the star's core using differential equations to understand its energy generation and evolution.

Case study Scenario

In the year 2032 A.D., a group of scientists led by Dr. Edward Astriak discovered a star in the main sequence phase of its evolution. The team has successfully designed a set of equipment to study the nuclear reactions taking place in the star's core. They want to determine how the abundance of helium (Y_{He}) in the core of Star SA-13 changes over time as a result of nuclear reactions. Specifically, the team is interested in the helium production rate through the proton-proton chain (PP-chain) responses.

Framework

Nuclear reactions occur in a star's core, where temperatures and pressures are incredibly high, generating the energy that allows stars to shine. The primary nuclear reactions in stellar cores involve fusing lighter elements into heavier elements, releasing tremendous amounts of energy. This process is known as nuclear fusion. Proton-Proton chain reaction is a type of fusion reaction which involves the following steps:

Step 1: Two protons (hydrogen nuclei) combine to form a deuterium nucleus (one proton and one neutron), releasing a positron (e+) and a neutrino (ν).

$H + H \rightarrow D + e^+ + \nu$

Step 2: A proton collides with a deuterium nucleus, creating a helium-3 nucleus and releasing a gamma ray (γ).

$H + D \rightarrow He^3 + \gamma$

Step 3: Two helium-3 nuclei combine to form helium-4 (alpha particle) and two protons.

$He^3 + He^3 \rightarrow He^4 + H + H$

Governing Equations

To model the change in helium abundance (Y_{He}) over time, the team employs the following differential equation based on the nuclear reaction rate:

$dY_{He} / dt = (2/m_p) \cdot R_{pp} - Y_{He} / t_{conv}$

Where:

- Y_{He} is the mass fraction of helium.
- m_p is the proton mass.
- R_{PP} is the rate of the first step of the PP-chain reaction.
- t_{conv} is the convective timescale (if convection is present).

The team calculates the value of R_{pp} using the reaction rate equation: $R_{pp} = n^2_p \cdot \langle \sigma v \rangle$

Where:

- n_p is the number density of protons.
- $\langle \sigma v \rangle$ is the reaction cross-section (σ) averaged over the relative velocity (v) of the colliding protons.

Data Collection

The team collects observational data from various sources, including stellar spectroscopy, solar observations, neutrino observatories like the Super-Kamiokande, and ground-based telescopes. These observations include composition, temperature, pressure, density measurements, and information on the type of nuclear reaction in the star.

Parameter Estimation

Through their observations, the team ascertained that the core of the star SA-13 is spherically symmetric. The group decided to emphasize the first step of the PP-chain reaction, which is the fusion of two protons to form deuterium, emitting a positron and a neutrino over the time scale of their analysis. Additionally, the team also estimates values of the following parameters by utilizing the collected data:

- The number density of protons (n_p) = 10^{31} cm^{-3}
- $\langle \sigma v \rangle = 1.0 * 10^{-25}$ cm^3/s
- Convective timescale (t_{conv}) = 10^7 s
- Mass of proton (m_p) = $1.6726219 * 10^{-27}$ kg

Application of the equations:

The team first uses the reaction rate equation to determine the value of R_{pp} as:

- *Calculation of R_{pp}:*
  $R_{pp} = n^2_p \cdot \langle \sigma v \rangle$
  $R_{pp} = (10^{31}\ cm^{-3})^2 \cdot (1.0 * 10^{-25}\ cm^3/s)$
  $R_{pp} = (10^{37}\ m^{-3})^2 \cdot (1.0 * 10^{-31}\ m^3/s)$
  $R_{pp} = 10^{43}\ m^{-3}s^{-1}$
  So, the value of the rate of the first step of the PP-chain reaction is approximately $10^{43}\ m^{-3}s^{-1}$.

Now,

- *Simplifying the differential equation:*
  $dY_{He} / dt = (2/m_p) \cdot R_{pp} - Y_{He} / t_{conv}$
  Substituting the values:
  $dY_{He} / dt = (2/1.6726219 * 10^{-27}\ kg) \cdot (10^{43}\ m^{-3}s^{-1}) - Y_{He} / 10^7\ s$
  $dY_{He} / dt = 1.195 * 10^{70} - Y_{He} / 10^7\ s$
  This differential equation describes how the helium abundance (Y_{He}) changes over time in the core of Star SA-13 due to nuclear reactions and convective mixing.

The team discovers that at time t = 0, Y_{He} (t = 0) = 0.01, which means that initially, 1% of the core's mass is helium.

- *Further Simplification*
  $dY_{He} / dt = 1.195 * 10^{70} – 10^{-7} \cdot Y_{He}$
  The team uses the separation of variables to solve this first-order ordinary differential equation. The solution attains the form:
  $Y_{He}(t) = (1.195 * 10^{70}) / 10^{-7} + C \cdot e^{t * -10 \wedge -7}$
  Using the initial condition Y_{He} (0) = 0.01, we can solve for the constant C:
  $0.01 = 1.195 * 10^{77} + C \cdot e^{0 * -10 \wedge -7}$
  $0.01 = 1.195 * 10^{77} + C$
  $C = 0.01 - 1.195 * 10^{77}$
  So, the solution reduces to:
  $Y_{He}(t) = 1.195 * 10^{77} + (0.01 - 1.195 * 10^{77}) \cdot e^{t * -10 \wedge -7}$

The team now evaluates the final value of $Y_{He}(t)$ as t approaches infinity.
$Y_{He} (\infty) = 1.195 * 10^{77} + (0.01 - 1.195 * 10^{77}) \cdot 0$
$Y_{He} (\infty) = 1.195 * 10^{77} + 0$
$Y_{He} (\infty) = 1.195 * 10^{77}$

It means that in the long-term evolution of the star, the helium abundance in the core will approach a value of $1.195 * 10^{77}$. This value represents the equilibrium abundance of helium reached through nuclear reactions in the star's core.

Cosmological Implications

The study provides insights into the evolution of stars, particularly during their main sequence phase. Understanding how nuclear reactions lead to the gradual conversion of hydrogen into helium is fundamental in explaining how stars evolve and maintain their energy output. Moreover, understanding nuclear reactions in the core of a star is a fundamental building block for the universe's chemical composition.

Conclusion

In this case study, we delved into modeling nuclear reactions in a hypothetical star, Star SA-13, into tracking the evolution of helium abundance within its core. Utilizing fundamental principles of nuclear physics and astrophysics, we unveiled several critical insights into the life cycle of stars and the broader implications for our understanding of the cosmos as it deals with stellar evolution, nucleosynthesis, and energy generation in stars.

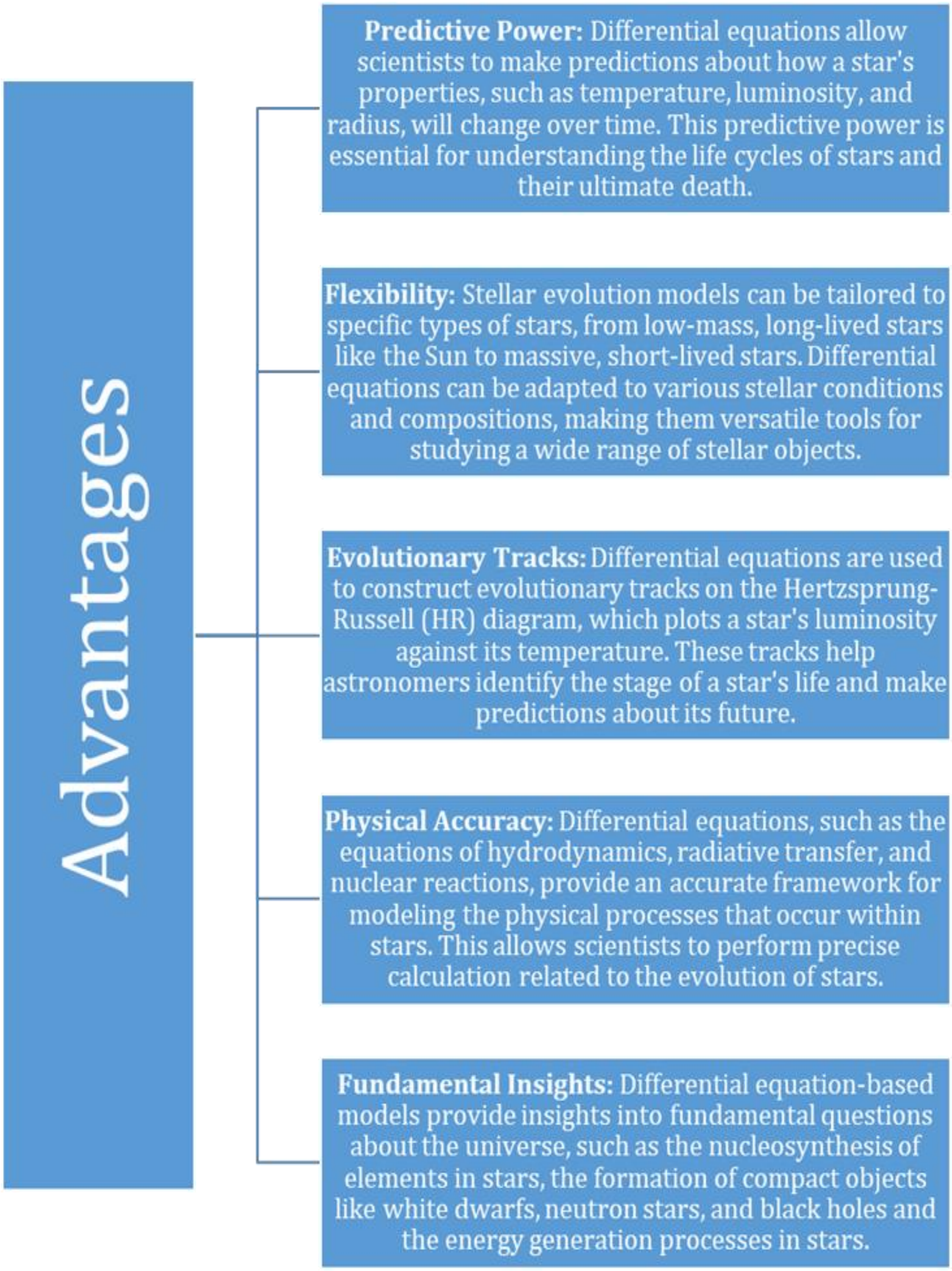
Advantages of using Differential Equations in Stellar Evolution

Differential equations play a crucial role in stellar evolution, which studies how stars change over time, from their formation to their eventual demise. Some of these advantages are listed below:

1. Predictive Power
2. Flexibility
3. Evolutionary Tracks

4. Physical Accuracy
5. Fundamental Insights

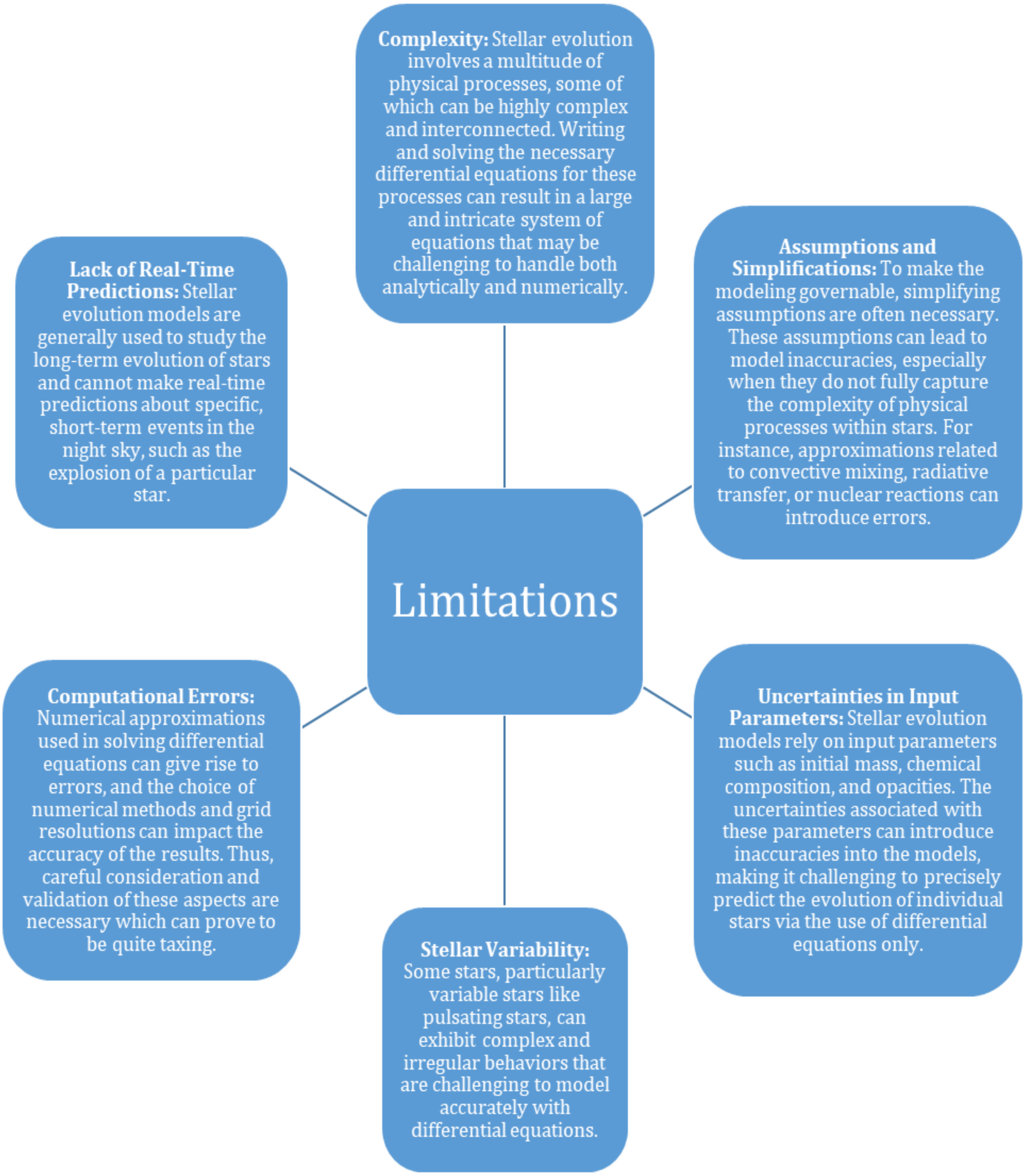
A chart explaining these advantages in further detail has been included below.



Picture 2. Advantages of Differential Equations

Limitations of using Differential Equations in Stellar Evolution

While differential equations are an indispensable tool in the study of stellar evolution, there are some drawbacks to their utility in this field.



Picture 3. Limitations of Differential Equations

Though essential, differential equations are not without flaws in their application in stellar evolution. To overcome these limitations, researchers can employ strategies and approaches, such as utilizing improved numerical methods, increasing the observational data, performing calibration and validation, and using high-performance computing and parallelization.

Conclusion and Suggestion

This report examines the differential equations associated with stellar evolution. Two case studies demonstrate the efficacy of these equations in modeling the internal and external processes taking place in a star. In contrast, the first case study models the energy transport within an evolving star. It discusses the two modes of energy transport in a star, namely, radiative energy transport and convective energy transport. Applying the differential equations associated with these modes of transport, the findings are as follows: 1) The luminosity due to radiative energy transport (Lr) is approximately 4.98 * 10^{28} J/s. 2) luminosity due to convective energy transport (Lc) is approximately 2.51 * 10^{16} J/s. Analyzing the magnitudes of both the above luminosities, it is clear that radiative energy transport is much more dominant than convective energy transport in the hypothetical main-sequence star S-T12. It leads to the conclusion that the S-T12 is a high-mass star with an extremely high core temperature.

Correspondingly, the second case study illustrates the utility of differential equations in modeling nuclear reactions taking place within a star's core. In this case study, the associated differential equation is applied to examine the change in Helium abundance in the core of a star, primarily through PP-chain reactions. The differential equation is solved by separating variables to calculate Helium's quantity when time approaches infinity. As per the calculations, the Helium abundance in the star's core is approximately 1.195*10^{77} as time approaches infinity. It clarifies that as the lifetime of the hypothetical star, SA-13 approaches infinity. As the hydrogen fusion reactions taking place in its core are completed, the core's Helium abundance will increase significantly, which will eventually trigger events such as core contraction and increase in temperature, shell burning and expansion, initiation of helium fusion and multiple fusion shells. These events in a massive star such as SA-13 can ultimately lead to a supernova explosion, leaving remnants like neutron stars or black holes.

This report illustrates how differential equations are used in stellar evolution. As such, simple ODEs have been used, and many data have been assumed; still, this report provides a new perspective on the applications of differential equations in real life. Further research could be conducted using this paper as a base and expanding the idea to a much broader range, utilizing the much more complex PDEs to design further the various structures and property profiles of a star.

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