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Mathematical Olympiad issues to identify students' reasoning ability using Polya's model

Mohammad Tohir[1*], Muhasshanah Muhasshanah[2], Riyan Hidayat[3], Erik Valentino[4], Tommy Tanu Wijaya[5]
[1*]Mathematics Education, Universitas Ibrahimy, Situbondo, East Java 68374, Indonesia
[2]Information Technology, Universitas Ibrahimy, Situbondo, East Java 68374, Indonesia
[3]Institut Penyelidikan Matematik & Department of Science and Technical Education, Universiti Putra Malaysia, Selangor 43400, Malaysia
[4]Mathematics Education, Sekolah Tinggi Keguruan dan Ilmu Pendidikan (STKIP) Bina Insan Mandiri, East Java 60229, Indonesia
[5]Mathematics Education, Beijing Normal University, Beijing 100875, China
[1*]matematohir@ibrahimy.ac.id, [2]muhasshanah@ibrahimy.ac.id, [3]riyanhidayat@fsmt.upsi.edu.my, [4]erikvalentino@stkipbim.ac.id, [5]tanuwijayat@gmail.com

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*Corresponding author

***Abstract*:**
This research aims to describe the level of mathematical reasoning ability of students in solving mathematical Olympiad problems based on problem-solving of the Polya model. This study employed descriptive analysis with a qualitative approach. Data were collected by using observation, documentation, and interviews. The study subjects were 27 junior high school students participating in the National Science Competition in Indonesia. Meanwhile, the Miles and Huberman analysis model was used as the data analysis. The results of this study indicated that: (1) the level of students' mathematical reasoning-ability based on the problem-solving of Polya models in the category of "*sufficiently competent*" (*high-group students*), in the category of "*less competent*" (*medium-group students*), and in the category of "*incompetent*" (*low-group students*); (2) the most complex and rarely performed stages by students in Polya's model were at the "*devising a plan*" and "*looking back*" stages; and (3) the Polya's model used in solving mathematical Olympiad test items was more suitable for those considered as routine-questions, and it was not suitable for non-routine questions. This study also showed that, on average, the students had difficulty finding initial ideas to start working on the test items.

Keywords: Mathematical Olympiad; Polya's Model; Problem-Solving; Reasoning Ability.

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Introduction

Students' mathematical reasoning ability needs to be continuously sharpened and improved to make some progress and reach a higher level. One's ability to use mathematical reasoning is closely related to achieving an inevitable conclusion based on what is expected from direct and indirect statements. Effective representation is the starting point of mathematical reasoning (Widjajanti et al., 2019; NCTM, 2009). Surajiyo and Andiani said that reasoning is the most general concept, which refers to one of the thought processes used to arrive at conclusions as a new statement from several other known ideas (Surajiyo & Andiani, 2007). Daldiyono defines reasoning as a person's thought process in drawing an inevitable conclusion through knowledge. Therefore, it can be said that mathematical reasoning demands one's ability to think logically and systematically and is a supreme mathematical cognitive domain (Tohir, 2017).

According to Sumarmo, one's mathematical reasoning ability can be divided into eight indicators: (1) drawing logical conclusions; (2) explaining with models, facts, properties, and relationships; (3) estimating answers and solution processes; (4) using patterns and relationships to analyze mathematical situations; (5) constructing and studying conjectures; (6) formulating the opposite following the rules of inference, checking validity argument; (7) constructing valid arguments; and (8) constructing direct, indirect proof, and using mathematical induction (Sulistiawati & Fatimah, 2016; Ayal et al., 2016; Sumartini, 2015). Tohir strengthens the statement by saying that the ability of mathematical reasoning is a process of thinking logically and mathematically in concluding either inductively or deductively based on the knowledge that has been obtained previously or unexpectedly in finding a valid truth (Tohir, 2017).

In this case, thinking logically about what is being thought over to find a particular answer is closely related to one's awareness of the ability to develop specific ways to solve mathematical problems. A particular problem is said to be a mathematical problem when it contains mathematical concepts in which the solution requires an indirect way. For instance, the question or problem in the form of a story, problems that need illustrations to be resolved, a case problem, an Olympiad problem, a puzzle, and others. Ruseffendi defines a problem in mathematics as a problem that students themselves can solve without using routine methods or algorithms (Muttaqin et al., 2021). According to Schoenfeld, problem-solving requires understanding the problem situation and the tools needed to make decisions, which leads to the individual's understanding (Yee & Bostic, 2014). According to Polya, problem-solving is an attempt to find a way out of a difficulty to achieve an objective that cannot be reached immediately (Pathuddin et al., 2022). Thus, someone's experience gained previously will affect the performance in solving problems faced by students.

Overall, mathematical problems that require specific strategies and techniques are mathematical Olympiad problems. According to Wiworo, the mathematical Olympiad questions have non-routine characteristics that require high school-level mathematics knowledge but involve advanced mathematical maturity (insight, accuracy, foresight, ingenuity, and experience) (Tohir, 2017). Maswar suggests that instructors need strategies in the learning process to solve various learning problems in mathematics so that the students are pleased and active and do not feel pressured to engage in the process of teaching and learning mathematics in class (Maswar, 2019). Their tactics varied according to their general ability; the more strategies they used to solve issues, the better their analogical reasoning became (Lailiyah et al., 2022). Meanwhile, according to Alford and Head (2017), several strategies can be used and may be very useful to solve a problem, especially questions that seem pretty complicated. Some methods discussed are looking for patterns, making drawings, writing and choosing notations, dividing cases, and working backward. Tohir proposed that the initial steps that must be owned and developed by Olympiad's participants in solving problems include the process of understanding the questions, which is understanding what is known and asked, then thinking of a strategy followed by implementing the plan and re-checking the completion procedures that have been done, thinking of the second strategy when the first one fails, reviewing the answer again (after successfully getting the answer) to get a valid response (Tohir, 2017).

The problem-solving stages used in this study are the Polya problem-solving model: understanding the problem, devising a plan, carrying out the project, and looking back. Polya's problem-solving model is expected to measure students' mathematical reasoning ability in solving Mathematical Olympiad questions (Hulaikah & Degeng, 2020). Hence, based on the description above, it is necessary to conduct specific research so that the students have appropriate mathematical reasoning abilities to solve mathematical Olympiad questions.

The results of research conducted by Sumartini found that the mathematical reasoning ability of students who got problem-based learning was better than students who got conventional knowledge (Sumartini, 2015). The research results by Tohir show that students' understanding of all matter mathematics Olympiad was still impoverished and needed guidance in the intensive continuously by using various development models (Tohir et al., 2018). The research results conducted by Arnellis et al. showed that the results of the post-test were better than the pre-test results in the coaching process of improving mathematics teachers' competence in compiling problems of the Mathematical Olympiad based on high-level thinking skills in junior high schools (SMP) in Pesisir Selatan Regency, West Sumatra, Indonesia (Arnellis et al., 2018). Therefore, it is necessary to have follow-up research on students who participate in the Mathematical Olympiad learning to get a proper learning outcome. Thus, the purpose of this study is to describe the level of students' mathematical reasoning ability in solving mathematical Olympiad questions based on Polya's solving-problem models.

Research Methods

Research design

This particular research used a descriptive type of research with a qualitative approach. The Research data were collected by using documentation, observation, and interviews. Then, the data is described and tested according to the theories. Based on the method, this research is called qualitative research. Qualitative research has the characteristics of having a natural background, having humans as a tool or instrument, using qualitative methods, analyzing the data inductively, compiling theories based on the data, having descriptive data, focusing more on the process than the results, having limitation determined by focus, comprising specific criteria for data validity, containing provisional design, and covering research results which are joint decisions or conclusions (Tohir et al., 2020). According to Lambert and Lambert, qualitative descriptive studies are the most "theoretical" of all qualitative research approaches (Lambert & Lambert, 2012). Besides, qualitative descriptive research is the least burdened study compared to other qualitative methods by pre-existing theoretical or philosophical commitments.

Participants

The subjects used in this study were 27 students of the Mathematical Olympiad class Junior High School students participating in the National Science Competition Indonesia. The indicators of students' mathematical reasoning abilities in this study were those of students' mathematical reasoning levels, which can be categorized into five categories: highly competent, competent, sufficiently competent, less competent, and incompetent, based on Polya's problem-solving model. The categorization of the ability levels helps predict students' mathematical reasoning abilities, especially in the field of mathematics. Indicators of student problem-solving success are divided into three categories: high, medium, and low. In comparison, the criteria for the level of mathematical reasoning ability in this study are presented in Table 1.

Table 1. Indicators of Students' Mathematical Reasoning Ability Levels

Category	Indicators of Students' Mathematical Reasoning Ability Levels
Level 0 incompetent	All indicators of students' mathematical reasoning are not fulfilled clearly or incompletely based on the four stages of problem-solving in the Polya model.
Level 1 less competent	All indicators of student mathematical reasoning are less or incompletely fulfilled based on the four stages of problem-solving in the Polya model.
Level 2 Sufficiently competent	All indicators of students' mathematical reasoning are sufficiently fulfilled clearly or completely enough based on the four stages of problem-solving in the Polya model.
Level 3 competent	All indicators of students' mathematical reasoning are fulfilled clearly or completely based on the four stages of problem-solving in the Polya model.
Level 4 very competent	All indicators of students' mathematical reasoning are fulfilled clearly or completely based on the four stages of problem-solving in the Polya model.

Source: adaptation from Mohammad Tohir (Tohir, 2017)

Data collection

The data collection techniques used to collect research data on the subject were competitions, problem-solving tests, documentation, observation, and interviews. Mathematical Olympiad test questions and interview guidelines had been validated by two lecturers at the Mathematics Education Study Program of the University of Jember, two lecturers at the Mathematics Study Program of the University of Ibrahimy Situbondo, and a lecturer at the Mathematics Education Study Program of the State University of Malang. The tests and interviews were conducted to get valid data. Tests and interviews were given to students who took part in the coaching class of the Mathematical Olympiad to gather data about the student's level of mathematical reasoning ability regarding the primary material within the Mathematical Olympiad itself. Then, the data that had been obtained were reduced, presented, concluded, and verified. Data verification was done by using the triangulation method. The data analysis techniques used in this study included (1) analyzing each level of students 'mathematical reasoning abilities based on the four steps of Polya's problem-solving model; (2) grouping the results of data analysis of students' mathematical reasoning levels; and (3) analyzing the factors which can affect the level of students' mathematical reasoning ability.

Data analysis

Qualitative research can present data through brief descriptions, charts, relationships between categories, flow diagrams, and so on. The data analysis technique used in this study is the flowchart presented by Miles and Huberman (1994). Narrative text has been the most frequently used form of data presentation for qualitative research data (Grossoehme, 2014). The data presentation includes classifying and identifying data and writing organized and categorized data sets to conclude. The conclusions obtained serve as supported data for conducting further research.

Research Results

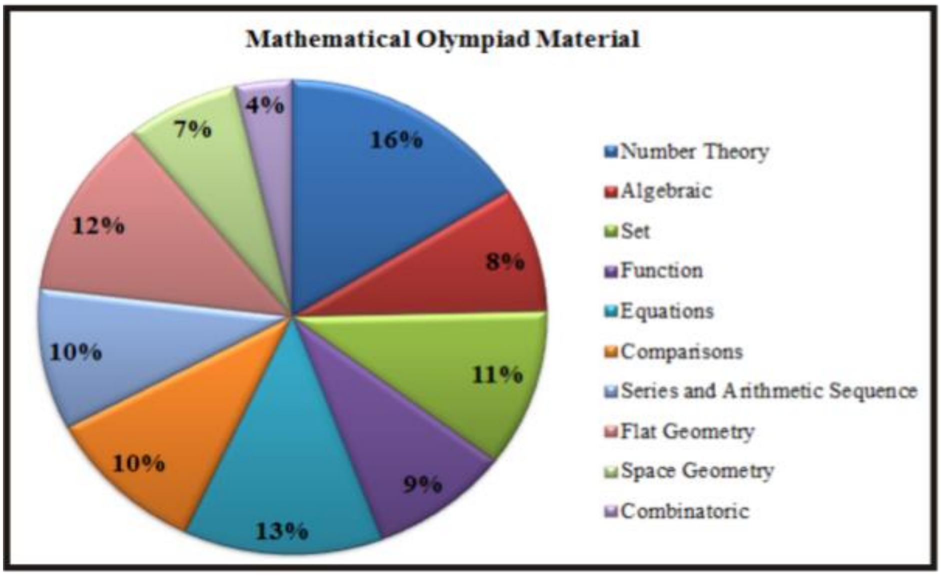
The results of the mathematics competition test are described based on the problem-solving of the Polya Model. Then, the success of problem-solving indicators was analyzed for the high, medium, and low group students, as seen in Table 2.

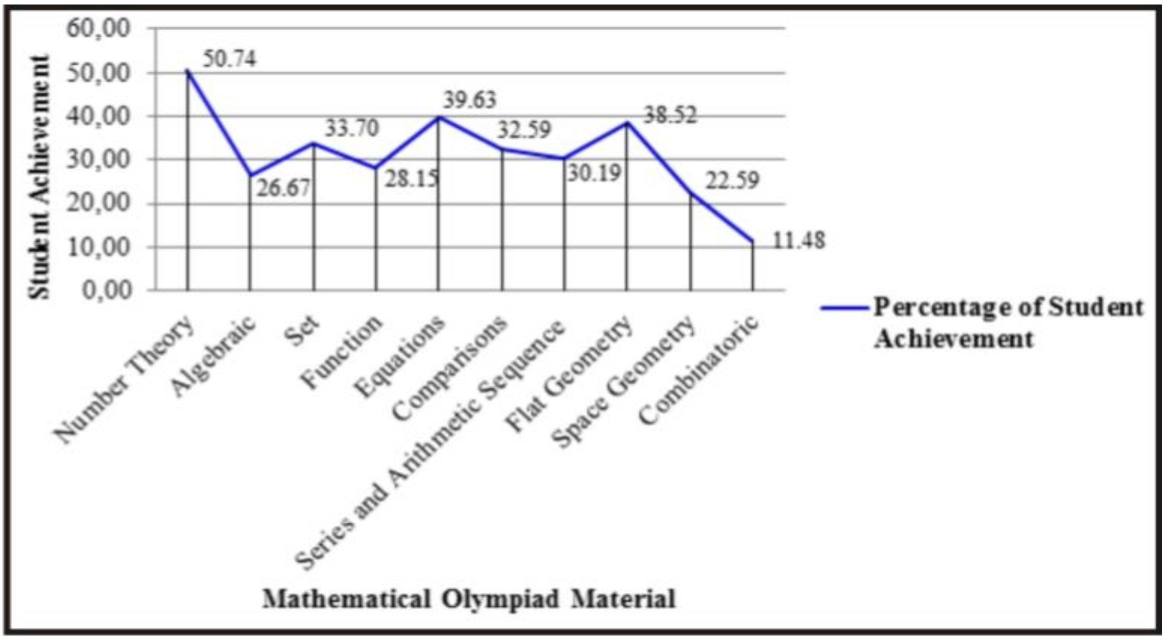
Table 2. The Results of The Mathematical Olympiad Test

Categories	N	Minimum	Maximum	Sum	Mean		Std. Deviation	Variance	Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic	Statistic	Std. Error
High	9	34.00	76.00	443.00	49.222	5.12287	15.36862	236.194	0.155	1.400
Medium	9	24.00	31.00	248.00	27.556	0.86781	2.60342	6.778	-1.845	1.400
Low	9	6.00	24.00	168.00	18.667	2.03443	6.10328	37.250	0.992	1.400

Table 2 shows that the test scores for high, medium, and low subjects are 49.22, 27.56, and 18.67. From these data, it is explained that there are differences in scores between the three subject groups, namely 21.67 and 8.89. The results of this test indicate that the ability between subject groups is significantly different. The third data obtained by researchers is the result of observations of student and teacher activities. The following shows the findings of the answers to the test questions for the student category groups.

The test results given to students can provide the data related to their difficulty completing the test items themselves. Hence, it is necessary to hold a continuous and unremitting learning process of the mathematical Olympiad test items typed for all students. This is due to the overall analysis results obtained in Picture 1.





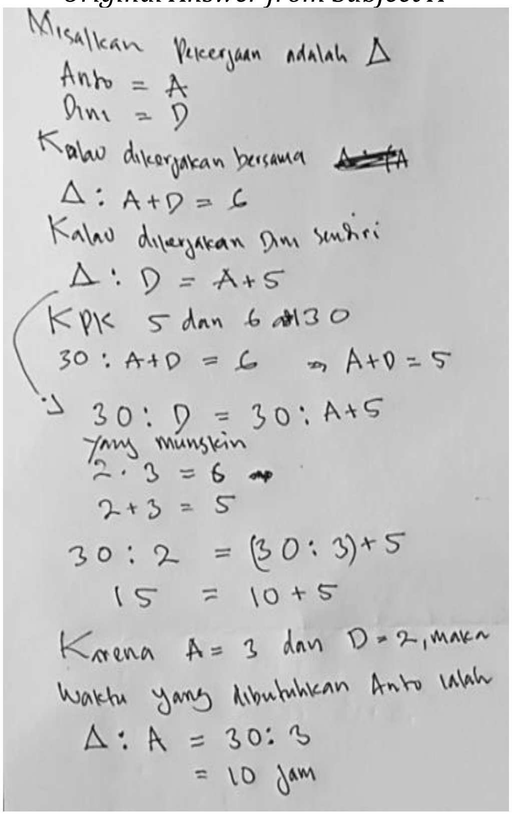
Picture 1. Percentage of Mathematical Olympiad Test Achievement Results

Based on Picture 1 above, it shows that the achievement of 27 student test results in all mathematical Olympiad material for the Junior High School level is very alarming and still far from what is expected by the institution, this is due to the fact that: (1) there were as many as 50.74% students who had mastered the competitions material on the subject matter of numerical theory; (2) there were 26.67% of students who mastered the competitions material in the sub-subject matter of algebraic forms; (3) there were as many as 33.70% of students who had mastered the mathematical Olympiad material on the sub-subject of sets' material; (4) there were only 28.15% of students who had truly mastered the competitions material on the sub-subject matter of function; (5) there were 39.63% of students who had mastered the competitions material on the sub-equality material; (6) there were 32.59% of students who had mastered the competitions material on the comparative subject sub-topic; (7) there were as many as

38.19% of students who had mastered the competitions material on the sub-subject material of series and arithmetic sequence; (8) there were 38.52% of students who had mastered the competitions material of flat geometry sub-subjects; (9) there were 22.59% of students who had mastered the competitions material on the sub-subject of space geometry; and (10) there were only 11.48% of students who had mastered the competitions material on the combinatoric sub-topics. Therefore, conducting a Mathematical Olympiad coaching, various learning techniques, and some test items that serve as continuous practice is necessary. The sub-topics of the competitions that needed special attention were sets, functions, arithmetic sequences, Geometry, and combinatorics. The following is an example of the analysis results of the test items given to students who took mathematical Olympiad coaching, junior high school students participating in the National Science Competition, Indonesia.

Student Answer A (the High-Group Students)

Original Answer from Subject A


Misalkan Pekerjaan adalah Δ
Anto = A
Dini = D
Kalau dikerjakan bersama A
Δ : A+D = 6
Kalau dikerjakan Dini sendiri
Δ : D = A+5
KPK 5 dan 6 30
30 : A+D = 6 → A+D = 5
30 : D = 30 : A+5
yang mungkin
2 · 3 = 6
2+3 = 5
30 : 2 = (30 : 3)+5
15 = 10+5
Karena A = 3 dan D = 2, maka
waktu yang dibutuhkan Anto ialah
Δ : A = 30 : 3
= 10 jam

Translation

Suppose the Job is Δ
Anto = A
Dini = D

If done together, then.
Δ : A + D = 6

If Dini Himself did it, then
Δ : D = A + 5

The LCM of 5 and 6 is 30
30 A + D = 6 and A + D = 5
30 + D = 30 : A + 5

so that maybe it is
2 . 3 = 6 and 2 + 3 = 5
30 : 2 = (30 : 3) + 5
15 = 10 + 5

Since A = 3 and D = 2, the time Anto will take is
Δ : A = 30 : 3
= 10 hours

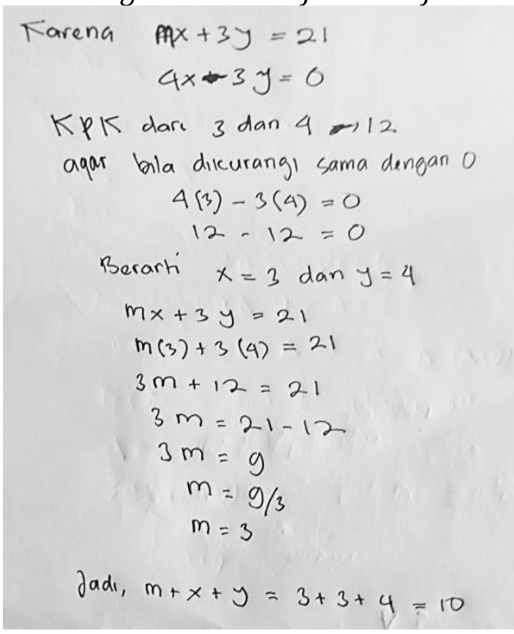
Picture 2. The Answer to Subject A

Based on answers from Subject A, it is known that the strategy used by Subject A was unique, engaging, and very different from the others. He was able to visualize the work symbolically by using the symbol Δ. The triangle symbol value is equal to 30. The resolving process on the importance of 30 is associated with the Least Common Multiple between 5 and 6. It is exciting and integrated, encouraging the researcher to learn more about the mathematical thinking process of subject A. The answers from subject A, which are unique and exciting, enable the researcher to discover how the mathematical thinking process works in choosing and using the preferred problem-solving strategy. The following are the results of the interview with subject A.

Researcher	:	Have you known about this question before?
Subject A	:	Yes, but it is not exactly like this.
Researcher	:	What information do you know about this question?
Subject A	:	Anto and Dini worked together for 6 hours to finish the work. If Dini only does it, it takes 5 hours slower than Anto.
Researcher	:	What is asked from the question?
Subject A	:	How long does it take for Anto to finish the work alone?
Researcher	:	How did you get the answer to that problem?
Subject A	:	To make it easier, I try to visualize the work as ΔAnto with the Letter A and Dini with the letter D. Then I look for the Least Common Multiple of 5 and 6, which is 30. Next, let's say Δ = 30.
Researcher	:	Why did you use the symbol Δ as a case in point and then replace it with a value of 30? Please explain it!
Subject A	:	Triangles crossed my mind when I saw the numbers 5 and 6 for the first time, and then the Least Common Multiple of 5 and 6 is 30. There are three numbers.
Researcher	:	Why is the value of D = 2 and A = 3? Is it because A + D = 5? What is your reason?
Subject A	:	Actually, there are many possible numbers to use. In my opinion, the most probable ones are the two numbers; if multiplied, we will have 6, and when they are added together, we will have 5. Next, the adjustment to be made is the mathematical similarity model, which is 30: D = 30: A + 5. The most appropriate number to fill it in is -> 30: 2 = (30: 3) + 5. The left and right joints are both 15. Thus, A = 3 and D = 2.
Researcher	:	Why did you put the brackets on the right segment when you didn't have them before?
Subject A	:	Well, I believe I was given the freedom to solve the problem. We didn't have any brackets before because it was still in the form of letters. After replacing A with number 3 and D with number 2, I prioritize the sum's division. I made some adjustments to the similarity model. For example, I take -> 30: 2 = 30: 3 + 5. If I give the brackets of 30: 2 = 30: (3 + 5), it will produce inequality, such as 15 = 37.5; they do not match, sir. Moreover, when I also take -> 30: 3 = 30: 2 + 5. If I give the brackets of 30: 3 = (30: 2) + 5, it will also produce an inequality of 10 = 20.
Researcher	:	Are you sure about your answer? Why do you think so?
Subject A	:	Yes, I'm sure, sir, because I have rechecked it repeatedly, and it is the correct answer for me.

The answer of Student B (the Medium Group Students)

Original Answer from Subject B



Translation

Because $mx + 3y = 21$

$4x - 3y = 0$

Then the LCM of 3 and 4 is 12,

so that if it is reduced, it is equal to zero

$4(3) - 3(4) = 0$

$12 - 12 = 0$

Means $x = 3$ and $y = 4$

$mx + 3y = 21$

$m(3) + 3(4) = 21$

$3m + 12 = 21$

$3m = 21 - 12$

$3m = 9$

$m = \frac{9}{3}$

$m = 3$

Thus, $m + x + y = 3 + 3 + 4 = 10$

Picture 3. The Answer to Subject B

Based on answers from Subject B, it is known that the strategy used by Subject B was unique and exciting; it was very different from the others. He could find the values of x and y by finding the Least Common Multiple from the coefficients of the equation model. The following are the results of the interview with subject B.

Researcher : Have you known about this question before?

Subject B : I have never seen this problem

Researcher : What information do you know about this problem?

Subject B : First, it is about a two-variable linear equation system. Second, this question asks for the sum of $m + x + y$ values.

Researcher : If that is the case, how is the process of getting the answers to these questions?

Subject B : the strategy is to look for the Least Common Multiple of 3 and 4 so that when they are substituted, it produces a similarity of 0. Then, I try substituting them into equation one to find the values of m for $x = 3$ and $y = 4$.

Researcher : Why must you look for the Least Common Multiple of 4 and 3?

Subject B : Well, when they are substituted in the equation of $3x - 4y = 0$, it completes the value, sir. For example, $x = 3$ means that $4 \times 3 = 12$, 4 times x equals 12. Thus, $y = 4$. Thus, 12 is the Least Common Multiple of 3 and 4.

Researcher : Why do you assume that the value of $x = 3$? Please describe it in more detail.

Subject B : If $3x - 4y = 0$, then three times x must equal four times y. Thus, we look for the multiples numbers of 3 and 4 that are the same to be reduced and produce zero. In which it is 3. The smallest number obtained is 12, but it does not have to be the smallest because it can be 24. However, I take the smallest one. Thus, the answer three times x is the number 12, and 4 times y is the multiple number of 12. Since only the various numbers of 12 can make $3x - 4y = 0$. So, it's proven that 4 (3) - 3 (4) = 0. Likewise -> 3 × 8 - 4 × 6 = 0, 3 × 12 - 4 × 9 = 0, and so on shows that the value of x is the multiple number of 4 and y is the multiples number of 3. It is the final answer. So, for the equation of $4x - 3y = 0$, the same thing applies to the equation $3x - 4y = 0$.

Researcher : Where does the value of m = 3 come from?

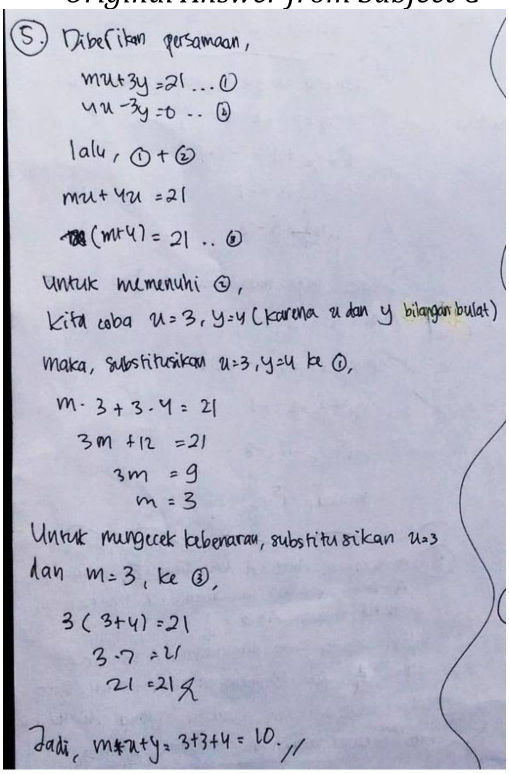
Subject B : well, I found the values of x and y (3,4), then I substituted them into the equation of $mx + 3y = 21$ so that it became m (3) + 3 (4) = 21 and the value of $m = 3$. So $m + x + y = 3 + 3 + 4 = 10$.

Researcher : Are you sure about your answer? Why do you think so?

Subject B : Yes, Sir, because according to my comprehensive knowledge, it's correct.

The following is an example of a discussion based on student C answers

Original Answer from Subject C



(5) Diberikan persamaan,
mx+3y=21 ...(1)
4x-3y=0 ..(2)
lalu, (1)+(2)
mx+4x=21
x(m+4)=21 ..(3)
Untuk memenuhi (2),
Kita coba x=3, y=4 (karena x dan y bilangan bulat)
maka, substitusikan x=3, y=4 ke (1),
m · 3 + 3 · 4 = 21
3m + 12 = 21
3m = 9
m = 3
Untuk mengecek kebenaran, substitusikan x=3 dan m=3, ke (3),
3 (3+4) = 21
3 · 7 = 21
21 = 21
Jadi, m+x+y = 3+3+4 = 10.

Translation

Given the equation,

$mx + 3y = 21$ ... (1)

$4x - 3y = 0$ ... (2)

Then, (1) + (2)

$mx + 4x = 21$

$(m + 4)x = 21$ ... (3)

To fulfill equation (2),

Let's try $x = 3$, and $y = 4$ (because x and y are integers)

So, substitute $x = 3$, and $y = 4$ to equation (1),

$m . 3 + 3 . 4 = 21$

$3m + 12 = 21$

$3m = 9$

$m = 3$

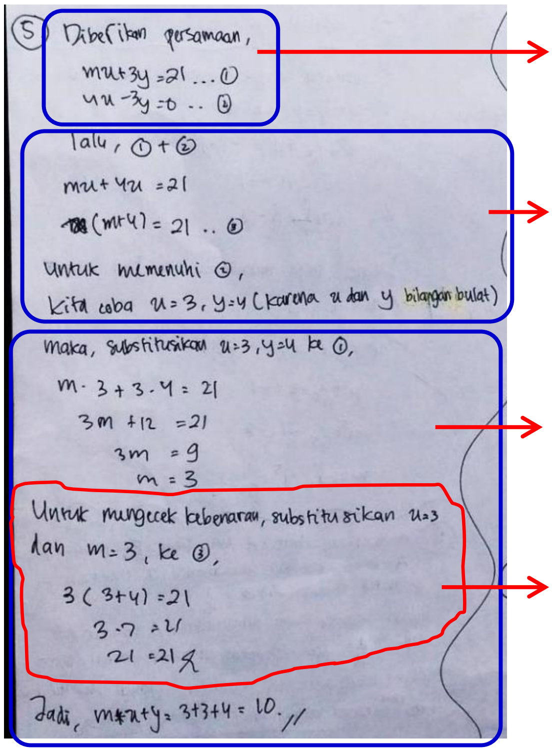
To check the truth, substitute the values for $x = 3$ and $m = 3$ into equation (3)

$3(3 + 4) = 21$

$3 . 7 = 21$

$21 = 21$

Thus, $m + x + y = 3 + 3 + 4 = 10$.



Step 1: the student had written what is known in the problem but had not written what is asked yet.

Step 2: The student had prepared a strategy, but it was flawed because the concept of factors in a number was not spelled out.

Step 3: Implementing the chosen strategy was based on the trial and error method.

Step 4: Even though the chosen strategy was flawed in concept, there was already a re-checking process to validate the answers obtained.

Picture 4. The Answer to Subject C

The analysis of Subject C answer shows that he had used the four stages of the Polya model. His level of mathematical reasoning ability was categorized within the "very competent" category. Besides the analysis results obtained within the stages of student C's answer, some other analysis results showed that all students did not do stage 4 on the seven test items. In comparison, for the other 3 test items, only a few students did not do the stage 4.

Students' Mathematical Reasoning Ability in solving Mathematical Olympiad problems.

Based on the overall analysis results, the high, medium, and low group students showed that they did not understand the Math competition material. It can be seen clearly from the average percentage of the Mathematical Olympiad test questions achievement, which was only 22.28%. Even though the three groups of students had two questions answered correctly, the concept used in applying the chosen strategy was not quite right, especially in question number 5. In this matter, within the value of $4x - 3y = 0$, they concluded that $x = 3$ and $y = 4$, which was not necessarily correct. The results of students' answers to question number 5 were 59.36%. Some of them were able to answer the problem correctly; they almost had the same remarks as the description of the answers within the three groups of students above. Therefore, it is still indispensable to hold continuous guidance both in a controlled and independent manner. The following is an example of the discussion results about the students' reasoning ability based on Polya's problem-solving model. The following in Table 3 presents the level of mathematical reasoning ability of students based on the problem-solving stages of the Polya Model.

Table 3. Level of Students' Mathematical Reasoning Abilities

Category	Model G. Polya
Level 0 (incompetent)	18.52%
Level 1 (less competent)	14.81%
Level 2 (sufficiently competent)	33.33%
Level 3 (competent)	22.22%
Level 4 (very competent)	11.11%

Based on the data in Table 3, the overall results indicate that students' mathematical reasoning ability level based on the steps within the Polya problem-solving model is "sufficiently competent." Meanwhile, based on the interviews and observations of each student of Polya's problem-solving model, it was found that the answers were the same, and they claimed that they had never been taught how to check the correct solutions for each problem faced. The stage within the Polya problem-solving model considered very difficult for students to experience is stage 2, the 'devising a plan' stage. Likewise, all of the students still had difficulty finding initial ideas to develop appropriate strategies for solving the problems faced.

Discussion

The lack of public understanding of mathematics's important role in everyday life impacts their perspective on mathematics. It is expected to say that mathematics is complicated, complex, tedious, and less valuable. When appropriately examined, within all levels of education, mathematics is taught to students with sufficient time. It is one of the proofs that mathematics is one of the most critical school subjects taught because it correlates with other subjects, and of course, it has great value for human life. It is not a matter of liking or dislike. Still, its convenience and relevance make it a compulsory subject at every level of education within the school system. Considering the benefits of mathematics that are so great in life, mathematics is regarded as a subject that is considered in the aspect of sustainability compared with other subjects.

In the learning process, mathematics must be taught with various techniques matching students' abilities. For students who love mathematics and have good mathematics skills, the learning process is given in the context of school and competitive classes. In this class, only students who possess high interest and enthusiasm for mathematics, both internally and externally, can participate. Furthermore, mathematical abilities also serve as a standard for the enrolment of this class. Why is that so? In this competition class, the process of learning mathematics itself is more directed to the more complex mathematical problems, which require critical reasoning and a high level of critical thinking to solve the problem. It aligns with 21st-century learning

(Munawwarah et al., 2020), which requires students to have competence in critical thinking, creative thinking, communication, and collaborative action. Although the students learn mathematics as well, there are still differences in the aspects of problems taught, which are far more complex than the mathematical problems solved in school classes in general.

Whatever it is, the most essential thing we need to understand is that learning places more emphasis on the progress of a process, not on the results. Maswar defines learning as a process to help students learn well (Maswar, 2019). Thus, it can be assumed that a good learning process in the mathematical Olympiad class can lead students to become champions in the mathematical Olympiad at the local, national, and international levels. In essence, the process and effort never deceive results. However, success is not solely dependent on this matter. Of course, many other factors need to be considered as well. It is in line with the findings of research conducted by McGee (2015), which showed that the guidance or assistance of mathematics teachers within the Mathematical Olympiad class must be appropriately managed and carried out continuously and persistently. Based on the findings obtained by Tohir when participating in the training of mathematics competitions Coaches as a speaker in Mojokerto, Blitar, and Madiun District, it was found that a module/teaching material used in the learning process of mathematics within the material for mathematical Olympiad class always needs to be refined as the learning materials and learning resources for students so that through this process of learning, it is expected that it will improve the atmosphere of student's competition in a trustworthy manner (Tohir et al., 2018). Therefore, a test with typed Mathematical Olympiad test items and some relatively new test items is necessary.

The interview results show that student A has a level of mathematical reasoning ability in the category of "very competent" because he used a unique strategy within the process of thinking and understanding the problem in solving the problem itself. The research results conducted by Wulantina concluded that students with high ability in the preparation stage can correctly identify the issue being asked appropriately and choose the information needed, and those who cannot solve the problem correctly (Muttaqin et al., 2021; Munawwarah et al., 2020). The research results conducted by (Surya & Putri, 2017) also showed that when applying ideas, students with high mathematical abilities do not make mistakes in problem-solving, and they are challenged to solve problems in various ways and answers. The accuracy of implementing the chosen strategy is obtained based on previous experience. It is under the statement proposed by Cropley and Urban, in which they said that the incubation stage is the stage at which students form the relationship of the completion ideas and those they have previously obtained (Cropley & Urban, 2000).

Another finding that shows students' unique answers is that student B has mathematical reasoning-ability in the medium category because the thinking process used in understanding the problem is the trial and error strategy. In solving the problem, he is less thorough and inaccurate because he forgot to write one step. The research conducted by Wulantina suggested that students within the medium category are those who try to dig up information about the problem. They can identify issues that are asked well but are less consistent in choosing the information needed and the ones that do not solve problems (Munawwarah et al., 2020). Likewise, the results of the study conducted by Defitriani stated that less creative students tend not to try to break away from the problem but try to think of the solutions to the issues at hand (Sari et al., 2017). Meanwhile, based on a hypothetical theory developed by Siswono and Kurniawati, less creative students tend not to check the answers after completing the task (Tohir, 2017; Kadir, 2018).

Based on the results of the interview with subject B, it was found that subject B did not have a complex strategy for solving the Mathematical Olympiads' problem. He tends to use trial and error strategies until he can find the results of the solution that is considered the correct answer. In addition, he was also not being careful and thorough in choosing the steps in the problem-solving, so there were some missing steps in his answer. Thus, based on the existing indicators, subject B can be categorized as a subject with a level of reasoning ability in the medium category. It is relevant to the results of research conducted by Wulantina, which stated that students within the medium class are those who try to dig up information about the problem; they can identify issues that are asked well, but they are less consistent in choosing the information needed and the one which is not in solving the problems (Munawwarah et al., 2020).

Moreover, according to Su et al. (2023), one can forget the information obtained because he failed to change short-term memories into long-term memories due to lack of repetition or because he cannot group the information he received. Meanwhile, based on the hypothetical theory proposed by Siswono, less creative students tend not to check their answers after completing a particular task (Ursulasari, 2019). It is in line with the results of research conducted by Runco et al., which showed that students have creative potential, as evidenced by their creative activities and achievements outside of school. Still, this potential is not displayed when they are in school because there are usually more structures and restrictions in schools, and creativity requires autonomy and independence (Runco et al., 2017). Limitations and future research are discussed.

Based on the analysis of student C answers, it turns out that the answers were the same, and they claimed that they had never been taught how to check the correct solution for each problem. At the same time, the stage considered very difficult for students to experience was stage 2, which is devising a plan. Moreover, all of the students still experienced difficulty in finding initial ideas as a step to develop appropriate strategies to solve the problems faced. According to Charles (Murni et al., 2013), the purpose of the mathematical problem-solving exercise is to (1) develop thinking skills; (2) develop the ability to choose and use problem-solving strategies; (3) develop attitude and confidence in solving problems; and (4) develop the ability of to monitor and evaluating their ideas to solve problems (Murni et al., 2013). It is in line with the formulation expressed by Artzt and Yaloz-Femia in which they suggested that reasoning is a part of thinking that belongs to the process of generalization and drawing valid conclusions about ideas and how those ideas are interrelated (Napitupulu et al., 2016).

The data presented in Table 3 support the previous study conducted by Tohir et al., which suggested that the Polya problem-solving model was very suitable for routine questions typed problems. In contrast, the Krulik-Rudnick model was more suitable for non-routine issues, but when all of the students implemented the steps within the Krulik-Rudnick problem-solving model, some of the steps were still missing, i.e., the stage on how to find an initial ideal strategy in dealing with Mathematical Olympiad questions (Tohir et al., 2018). The results of research conducted by Tambunan showed that learning through a problem-solving process was more effective than implementing scientific approaches to students concerning their mathematical abilities in communicating, creativity, problem-solving, and mathematical reasoning (Tambunan, 2019). The results of other studies obtained by Hughes et al. showed that most students were found to be inaccurate in finding the solution to their problems and communicating the minimum mathematical reasoning in their written expressions (Hughes et al., 2020). Results of the research obtained by Nadrah showed that students' thinking styles might change according to their problem (Nadrah et al., 2020). Besides, students were inclined to use general vocabulary rather than academic ones, and those who could provide visual representations were more likely to answer the problem accurately (Alfin & Fuad, 2019). Therefore, specific steps are needed to help students find initial ideas when dealing with Mathematical Olympiad questions.

Conclusion

Based on the research results and discussion described previously, it can be concluded that: (1) based on the test results obtained for all of the students, it is very necessary to have continuous mathematical Olympiad learning process with various learning strategies that can support the objective of the learning process itself based on the goal determined, which is getting a gold medal in the National Science Competition in mathematics studies; (2) based on the overall results of the interviews, it was found that in the process of completing the ten questions given to them, they were able to finish all of them because they had already known the questions before and had worked on them; (3) the level of mathematical reasoning ability of students based on the steps of Polya problem solving model is within the category of "sufficiently competent"; (4) students were find it very difficult to find the initial ideas in dealing with a mathematical Olympiad question, in which the initial idea itself is considered to be a step in developing the appropriate strategy for solving mathematical Olympiad question itself; and (5) Based on the results of the analysis and discussion within the students' level of mathematical reasoning ability based on Polya's problem-solving model, it was found that the steps of Polya's problem-solving model were not always suitable to be used to solve all of the Mathematical Olympiad question typed.

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