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Local edge (a, d) –antimagic coloring on sunflower, umbrella graph and its application

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Abstract:
Suppose a graph $G = (V, E)$ is a simple, connected and finite graph with vertex set $V(G)$ and an edge set $E(G)$. The local edge antimagic coloring is a combination of local antimagic labelling and edge coloring. A mapping $f : V(G) \to \{1,2,\ldots,|V(G)|\}$ is called local edge antimagic coloring if every two incident edges $e_1$and e_2, then the edge weights of $e_1$and e_2 maynot be the same, $w(e_1) \neq w(e_2)$, with $e = uv \in G, w(e) = f(u) + f(v)$ with the rule that the edges e are colored according to their weights, w_e. Local edge antimagic coloring has developed into local (a, d)-antimagic coloring. Local antimagic coloring is called local (a, d)-antimagic coloring if the set of edge weights forms an arithmetic sequence with a as an initial value and d as a difference value. The graphs used in this study are sunflower graphs and umbrella graphs. This research will also discuss one of the applications of local edge (a, d)-antimagic coloring, namely the design of the Sidoarjo line batik motif. The result show that $\chi_{le(3,1)}(Sf_n) = 3n$ and $\chi_{le\left(\frac{3n}{2},1\right)}(U_{m,n}) = m + 1$. The local (a, d)-antimagic coloring is formed into a batik motif design with characteristics from the Sidoarjo region.

Keywords: local edge (a, d) –antimagic coloring, sunflower graph, umbrella graph.

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Introduction

Graph theory was firstly introduced by a mathematician from Switzerland named Leonhard Euler in 1736. Graph G is defined as a set (V, E) that consists of a set of vertices and a set of edges called edges. The set of the vertex is denoted by $V(G)$ and the set of edges is denoted by $E(G)$ (Richard, 2009). The number of the vertex on a graph G is called the order denoted by $|V(G)|$ while the number of edges in a graph G is called the size denoted by $|V(G)|$ (Grifin, 2012). The degree of a graph G is the number of adjacent edges at a vertex in the graph. The greatest (maximum) degree of a graph G is denoted by $\Delta(G)$ and the smallest (minimum) degree of a graph G is denoted by $\delta(G)$ (Chartrand & Zhang, 2012).

There were various topic in graph theory such as graph labelling and graph coloring. Graph labelling was a mapping of the graph elements to the natural numbers. Graph labelling consisted of edges labelling, vertex labelling and total labelling. Anti-magic labels generate different edge weights denoted by $w(uv)$ (Guichard, 2022).

Graph coloring is giving color to the elements in a graph. Graph coloring has the condition that each neighboring element has a different color (Purwanto et al., 2006). Graph coloring consists of vertex coloring and edge coloring. Vertex coloring is the provision of color at vertex provided that neighboring vertex have different colors (Kristiana, Alfarisi, et al., 2022; Kristiana, Hidayat, et al., 2022). Edge coloring is the assignment of color to the edges of a graph provided that the neighboring edges have different colors. The minimum number of colors for coloring a graph G is called the chromatic number denoted by $\chi(G)$ (Chartrand & Zhang, 2008).

This research discusses one of the graph topic, namely local (a, d) –antimagic coloring. Local (a, d) –antimagic coloring is a development of local edge antimagic coloring. Local edge antimagic coloring is a combination of local antimagic and edge coloring. A mapping f: $V(G) \rightarrow \{1, 2, \ldots, |V(G)|\}$ is called local edge antimagic coloring if every two edges that are incident at the same vertex, e_1 and e_2, then $w(e_1) \neq w(e_2)$, which $e = uv \in G$, $w(E) = f(u) + f(v)$. By the rule that edges e are colored according to their weights, w_e. In local edge antimagic coloring, the minimum number of colors is denoted by X_{lea} (Agustin et al., 2018). The concept of local (a, d) –antimagic coloring has something in common with the concept of local edge antimagic coloring. Local antimagic coloring is called local (a, d) –antimagic coloring if there is a set of edge weights $w(e)$ forming an arithmetic sequence that has a start a and difference b. The minimum number of colors to color graph G or the local (a, d) –antimagic chromatic number is denoted by $\chi_{le(a,d)}G$.

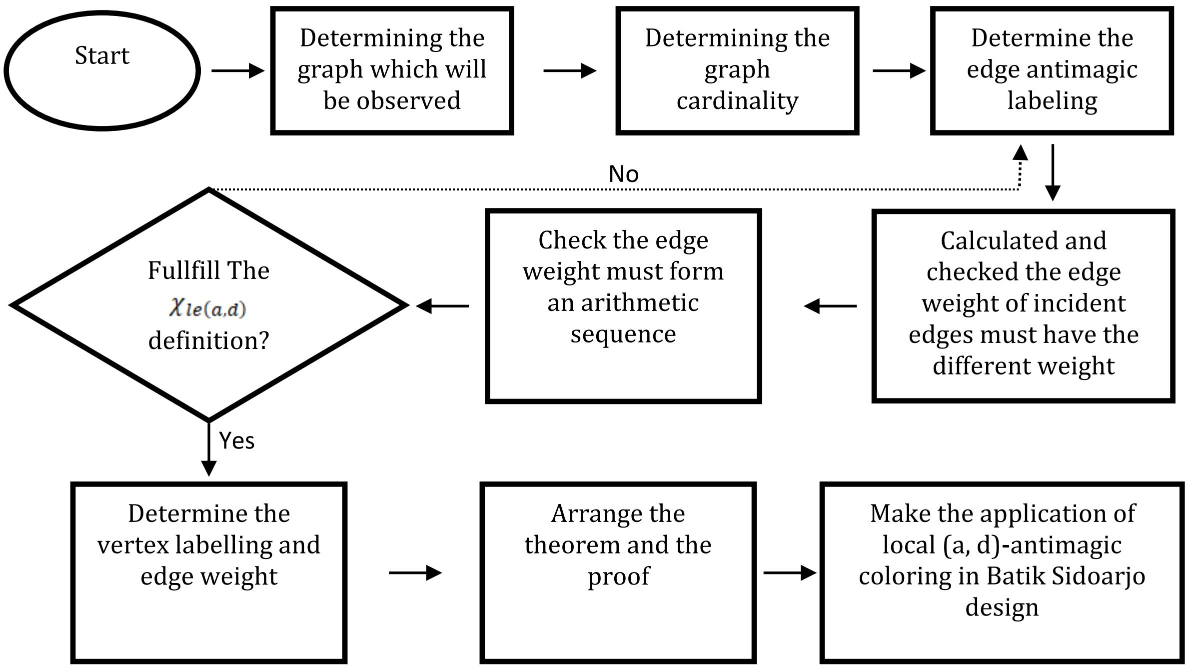
This is the development of local edge antimagic coloring of graphs study which had been observed by (Adawiyah et al., 2019; Agustin, 2017; I. Agustin & Dafik, 2022; Dafik et al., 2021). This research also the development of super (a, d) –antimagic labelling which have been observed by (Adawiyah et al., 2019; Agustin et al., 2017; Figueroa-Centeno & Muntaner-Batle, 2021; Ming-ju, 2013; Moussa & Badr, 2016; Prihandini & Agustin, 2018; Purnapraja et al., 2014). Another research about the labelling of graph have been observed by (Gallian, 2022; Agustin & Prihandini, 2019; Prihandini et al., 2018, 2020; Prihandini et al., 2019). Some previous research about this topic has been conducted by (Almaidah et al., 2022) with the title of her research is on local (a, d) –antimagic coloring of some specific classes of graphs. In this research, we investigate local (a, d) –antimagic coloring of umbrella graph and sunflower graph which have not been observed yet. We also define the application of this research in designing batik.

The concept of local (a, d) –antimagic coloring can be applied in creating batik designs (Rosen, 2012). Batik is one of Indonesia's unique products and a symbol of Indonesian culture. Batik is included in crafts derived from ancestors that have artistic value. Batik in Indonesia is very diverse and has an appeal or value contained in these motifs. The motif in batik is the main element and has a very important aspect. Batik motifs must be designed with attention to color thus they have good value. The coloring in this batik motif is designed symmetrically and regularly. With this, accuracy is needed in determining color choices. This color choice must also look elegant and aesthetic. In this paper, we use the concept of local (a, d) –antimagic coloring in making batik motif designs especially in Sidoarjo batik motif design.

Research Methods

The method used in this research is the deductive axiomatic method and pattern recognition (Kurniawati et al., 2021). The following is an explanation of the method: The axiomatic deductive method is a method that uses deductive proofs that use existing axioms, lemmas, and theorems to solve the problem of the topic to be studied. In this topic to determine or prove the local (a, d) –antimagic chromatic number $X_{le(a,d)}$ by using the lemma or theorem that has been studied previously. Pattern detection method (pattern recognition) is a method used to determine patterns in Local (a, d) –antimagic coloring so that a local (a, d) –antimagic chromatic number will be denoted by $X_{le(a,d)}$ (Adawiyah et al., 2019). Next apply local edge (a, d) –antimagic coloring to design batik.

This research is explorative and applied research (Adawiyah & Prihandini, 2022). The research procedure are as follow: 1. Start, 2. Determining the graph which will be observed, 3. Determining the graph cardinality, 4. Determine the edge antimagic labelling, 5. Calculated and checked the edge weight of incident edges must have the different weight, 6. Check the edge weight must form an arithmetic sequence, 7. Determine the vertex labelling and edge weight, 8. Arrange the theorem and proof of local (a, d) –antimagic coloring, 9. Make the application of locad (a, d) –antimagic coloring in Batik Sidoarjo design. In general, we can see the research procedure in Picture 1.



Picture 1. Research Procedure

Results and Discussions

In this section, we discussed some theorems related to the local edge (a, d) −antimagic coloring and its application. The theorems of local edge (a, d) −antimagic coloring of sunflower (Sf_n) graph and umbrella graph $(U_{(m,n)})$ are as follow.

Theorem 1 For every integer with $n \geq 4$, *local edge* (a, d) −*antimagic coloring* of the sunflower graph (Sf_n) is $\chi_{le(3,1)}(Sf_n) = 3n$.

Proof. A sunflower (Sf_n) graph is a graph with a set of vertices $V(Sf_n) = \{c, x_i, y_i, z_i; 1 \leq i \leq n\}$ and a set of edges $E(Sf_n) = \{cx_i, cy_i, cz_i, y_i z_i, z_i z_{i+1}; 1 \leq i \leq n\}$. The cardinality of the edge sets and vertex sets of a sunflower graph are $(Sf_n)$and $|E(Sf_n)| = 5n$, respectively $|V(Sf_n)| = 3n + 1$. In order to prove that *local edge* (a, d) −*antimagic coloring* $\chi_{le(3,1)}(Sf_n) = 3n$, we have to prove the upper and lower bounds of the local edge (a, d) −antimagic coloring on the sunflower graph, namely $3n \leq \chi_{le}(Sf_n) \leq 3n$.

First, we will analyze the lower bound of the *local edge* (a, d) −*antimagic coloring on the sunflower graph.* In order to show $\chi_{le} \geq 3n$, we analyze the maximum degree of the sunflower graph $\Delta(Sf_n)$. We know that $\Delta(Sf_n) = 3n$. Based on the concept of *local edge* (a, d) −*antimagic coloring, neighboring edges cannot have the same color so we get* $\chi_{le} \geq \Delta$. Based on this analysis, it is obtained $\chi_{le(3,1)}(Sf_n) \geq 3n$.

Next, we will prove the upper bound of the *local edge* (a, d) −*antimagic coloring* on the sunflower graph. The upper bound $\chi_{le} \leq 3n$ is proved through bijective mapping $f: V(Sf_n) \to \{1,2,3, \dots, |V(Sf_n)|\}$ as follows.

$$f(c) = 1$$
$$f(x_i) = 2n + i + 1$$
$$f(y_i) = 2n - i + 2$$
$$f(z_i) = i + 1$$

Based on the vertex labelling on the sunflower graph, the edge weights are obtained as follows:

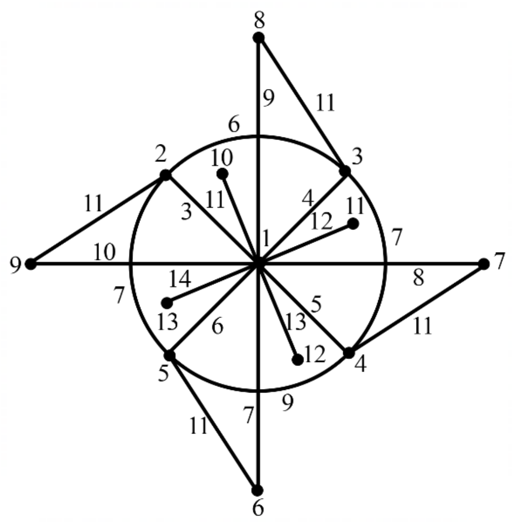
$$w(cx_i) = 2n + i + 2, \text{for} 1 \leq i \leq n$$
$$w(cy_i) = 2n - i + 3, \text{for} 1 \leq i \leq n$$
$$w(cz_i) = i + 2, \text{for} 1 \leq i \leq n$$
$$w(y_i z_i) = 2n + 3$$
$$w(z_i z_{i+1}) = 2i + 3, \text{for} 1 \leq i \leq n - 1$$
$$w(z_i z_{i+1}) = n + 3, \text{for} i = n$$

Based on the edge weights, the set of edge weights which obtained is $W = \{3,4,5,6, \dots ,3n + 1,3n + 2\}$. The smallest edge weight is obtained $a = 3$ by $d = 1$. Based on these edge weights, we get the cardinality of the set of edge weights as follows:

$$U_w = a + (|W| - 1)d \Leftrightarrow 3n + 2 = 3 + (|W| - 1).1 \Leftrightarrow |W| = 3n$$

Thus, we know that the minimum number of colors required is $3n$. Therefore, we obtain the upper bound of the *local edge* (a, d) *−antimagic coloring* on the sunflower graph $\chi_{le(3,1)}(Sf_n) \leq 3n$.

Based on the analysis of the upper and lower bounds of the *local edge* (a, d) *−antimagic coloring* on a sunflower graph, it is Sf_nobtained $3n \leq \chi_{le}(Sf_n) \leq 3n$. Thus, it can be concluded that$\chi_{le}(Sf_n) = 3n$. Figure 2 shows an example of *local edge* (a, d) *−antimagic coloring* on a sunflower graph Sf_4.



Picture 2. *local edge* (a, d) *−antimagic coloring* of *sunflower graph*(Sf_4)

Theorem 2. For every integer with $n \geq 3\ dan\ m \geq 3$, local edge (a, d) −antimagic coloring of an umbrella graph $(U_{m,n})$ is $\chi_{le\left(\frac{3n}{2},1\right)}(U_{m,n}) = m + 1$.

Proof. An umbrella $(U_{(m,n)})$graph is a graph with a set of vertices $V(U_{m,n}) = \{x_i; 1 \leq i \leq n\} \cup \{x_{i,j}; 1 \leq j \leq m\}$and a set of edges $E(U_{m,n}) = \{x_1 x_{1,j}; 1 \leq j \leq m\} \cup \{x_{1,j} x_{i,j+1}; 1 \leq j \leq m-1\} \cup \{x_i x_{i+1}; 1 \leq i \leq n-1\}$. The cardinality of the edge sets and vertex sets of the umbrella graph are $(U_{(m,n)})$ and $|E(U_{m,n})| = 2m + n - 2$, respectively $|V(U_{m,n})| = n + m$. To show local edge (a, d) −antimagic coloring $\chi_{le\left(\frac{3n}{2},1\right)}(U_{m,n}) = m + 1$, then we have to show $m + 1 \leq \chi_{le} \leq m + 1$.

First, we will prove the lower bound of the local edge (a, d) $-$antimagic coloring on an umbrella graph, namely $\chi_{le} \geq 3n$. To show $\chi_{le} \geq m + 1$, we analyze the maximum degree of an umbrella graph that is $\Delta(U_{m,n})$. We know that $\Delta(U_{m,n}) = m + 1$. Based on the concept of local edge (a, d) $-$antimagic coloring, adjacent edges cannot have the same color thus $\chi_{le} \geq \Delta$. Therefore, we get $\chi_{le} \geq m + 1$.

Then, we prove the upper bound of the local edge (a, d) $-$antimagic coloring on the umbrella graph, namely $\chi_{le} \leq m + 1$. We proof of the upper bound $\chi_{le} \leq m + 1$ through bijective mapping $f: V(U_{m,n}) \rightarrow \{1,2,3, \dots, |U_{m,n}|\}$ as follows:

$$f(x_i) = \begin{cases} \frac{n-i+2}{2}; for\ i\ odd\ and\ n\ odd \\ \frac{n-i+1}{2}; for\ i\ odd\ \text{and}\ n\ even \\ \frac{n+2m+i+1}{2}; for\ i\ even\ and\ n\ odd \\ \frac{n+2m+i}{2}; for\ i\ even\ and\ n\ even \end{cases}$$

$$f(x_{i,j}) = \begin{cases} \frac{2n+j+1}{2}; for\ \ j\ odd \\ \frac{2n-j+2}{2}; for\ j\ even \end{cases}$$

Then we will show that f is the local edge (a, d) $-$antimagic coloring of $(U_{m,n})$ with the edge weights as follows:

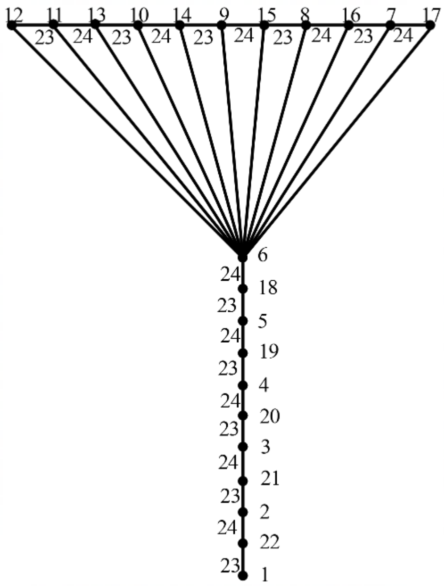
$$w(x_i x_{i,j}) = \begin{cases} \frac{3n+j+2}{2}; for\ j\ odd\ and\ n\ odd \\ \frac{3n-j+3}{2}; for\ j\ even\ and\ n\ odd \\ \frac{3n+j+1}{2}; for\ j\ odd\ \ and\ n\ even \\ \frac{3n-j+2}{2}; for\ j\ even\ \ and\ n\ even \end{cases}$$

$$w(x_{1,j} x_{1,j+1}) = \begin{cases} \frac{4n+2}{2}; for\ j\ odd \\ \frac{4n+4}{2}; for\ j\ even \end{cases}$$

$$w(x_i x_{i+1}) = \begin{cases} n+m+2; for\ i\ odd\ and\ n\ odd \\ n+m+1; for\ i\ even\ and\ n\ odd \\ n+m+1; for\ i\ \text{odd}\ and\ n\ even \\ m+n\ ; for\ i\ \text{even}\ and\ n\ even \end{cases}$$

Based on the edge weights, the set of edge weights obtained is $W = \{\frac{3n}{2}, \frac{3n+1}{2}, \frac{3n+2}{2}, \dots, m+n+2\}$. The smallest edge weight is obtained $a = \frac{3n}{2}$ by $d = 1$, we have $\chi_{le\left(\frac{3n}{2},1\right)}(U_{m,n}) \leq m+1$. It is concluded that $\chi_{le\left(\frac{3n}{2},1\right)}(U_{m,n}) \leq m+1$.

Figure 3 shows the local edge (a, d) −antimagic coloring on the umbrella graph $(U_{11,11})$



Picture 3. *local edge* (a, d) *−antimagic coloring of umbrella graph* $(U_{11,11})$

Application of local edge (a,d)-antimagic coloring

Batik is a cultural heritage from the ancestors of the Indonesian nation that combines art and technology. The most important part in batik is batik motifs with various forms. Batik motifs are made by paying attention to the elements in order to create beautiful and aesthetic batik. The elements of the batik motif itself, namely the coloring pattern, the design of the batik motif must be designed symmetrically, regularly, and pay attention to the use of color. Aesthetic batik motifs require accuracy in determining the type of color choice. Based on this, batik motif designs are represented in the form of graphs using the concept of local edge (a, d) –antimagic coloring with the STEM approach. Here is the process of STEM approach in designing batik using local edge (a, d) –antimagic coloring concept.

a. Science

We should design batik motifs by paying attention to the coloring patterns in order to create aesthetic batik motifs. Batik motif designs are represented in the form of graphs using the concept of local edge (a, d) –antimagic coloring.

b. Technology

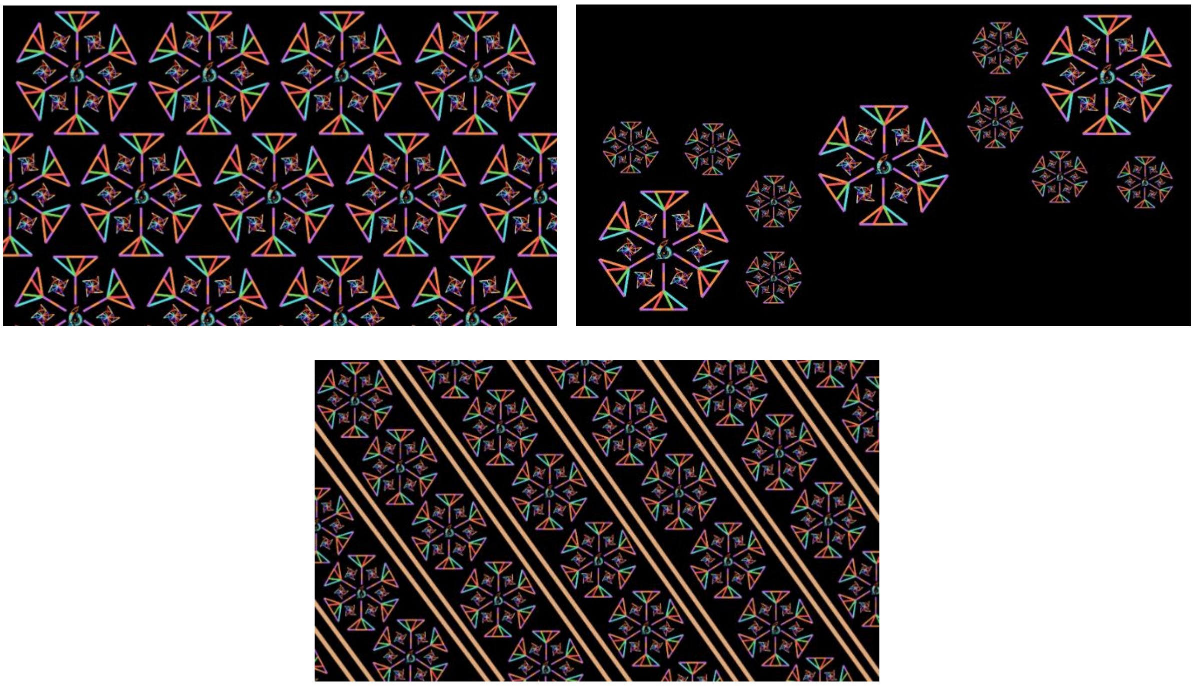
The FX Draw software is used to draw graphs, Corel Draw software is used to draw graph representations for batik motif designs. Photoshop CS5 software to color the graph representation. Meanwhile, the Canva software is used to combine graph representations to create Sidoarjo batik motif designs.

c. Engineering

Engineering techniques are used to solve the problem by applying the local (a, d)-edge antimagic coloring concept by representing batik motif designs in graph form.

d. Mathematics

The elements of mathematics in solving the problems related to the local (a, d)-edge antimagic coloring is the use of mathematical calculations in determining the permutation of the minimum color set in the striped batik motif design. The following are the results of making batik motif designs represented in the form of graphs using the concept of local edge (a, d) –antimagic coloring.



Picture 4. Sidoarjo batik motif design

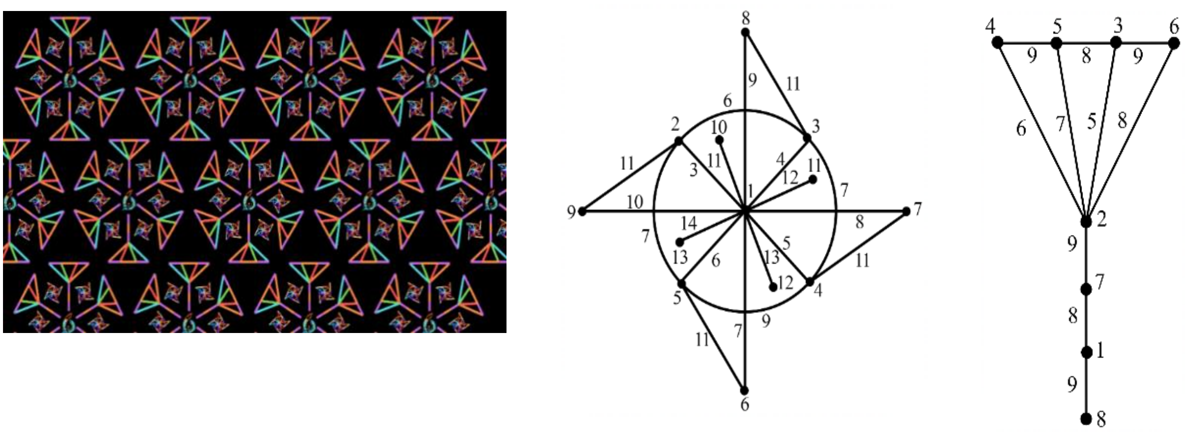
Based on the results in theorem 1 and theorem 2, we derived batik Sidoarjo motif design. In the Sidoarjo batik motif design, we combined between sunflower graph and umbrella graph. The pattern in derived between amalgamations of umbrella graph and between each umbrella graph, we add sunflower graph thus it become a good looking pattern. The color of each edge in umbrella graph and sun graph is defined by using local edge (a, d) –antimagic coloring. We gave some colors depend on the color that we got in theorem 1 and theorem 2. Through this process, we can get a new batik motif design with a beatiful and ellegant motif.

Discussion

In this paper, we have analyzed about local edge (a, d) –antimagic coloring of sunflower graph and umbrella graph. The results show two theorems about local edge (a, d) –antimagic coloring of sunflower graph and umbrella graph as follow:

Theorem 1 For every integer with $n \geq 4$, *local edge* (a, d) *–antimagic coloring* of the graph (Sf_n) is $\chi_{le(3,1)}(Sf_n) = 3n$.

Theorem 2. For every integer with $n \geq 3\ dan\ m \geq 3$, local edge (a, d) –antimagic coloring of an umbrella graph $(U_{m,n})$ is $\chi_{le\left(\frac{3n}{2},1\right)}(U_{m,n}) = m + 1$.



Picture 5. (a) Sidoarjo Batik Motif design, (b) local edge (a, d) –antimagic coloring of sunflower graph(Sf_4), (c) local edge (a, d) –antimagic coloring of Umbrella graph$(U_{4,4})$

Based on those two theorems, we applied in designing Sidoarjo Batik. We use Sunflower graf Sf_4 and Umbrella graph $U_{4,4}$ in designing Sidoarjo Batik Design. Based on theorem 1, and local edge (a, d) –antimagic coloring of sunflower graph (Sf_4) in figure 5(b), we got 12 different edge weight, which become colors in of Sf_4 such ad {3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14}. In Sidoarjo batik motif design, we convert the number to 12 certain color.We give the same color for the edge with the same weight. Thus we got the motif design of Sf_4 in figure 5(a). Same condition with Sf_4, in umbrella graph $U_{4,4}$, first we analyze the vertex label and the edge weigh of $U_{4,4}$. We can see the local edge (a,d)-antimagic coloring of Umbrella graph $(U_{4,4})$ in figure 5(c). We got the 5 edge weight in $U_{4,4}$ which is {5,6,7,8,9}. We convert these edge weights into 5 different color. After coloring the graph, then we combine Sf_4 and $U_{4,4}$ to get a new pattern in Sidoarjo Batik. We can see the combination of Sf_4 and $U_{4,4}$ in figure 4.

The results of this research is new result in local edge (a, d) –antimagic coloring study. This research is getting along with the previous research which have been conducted by (Almaidah et al., 2022). We applied the study of local edge (a, d) –antimagic coloring in real life apllication which is the strength of this research. Batik is one of Indonesian pride which can be the symbol of Indonesian culture. The study of batik design that come from mathematics application is relatively rare. Some other researcher who want do the research about this topic was really warmly welcome because this topic was relatively new topic in graph study. There are many other graphs which had not been observed yet.

Conclusions and Suggestions

Based on the results of the discussion, the theorem of sunflower (Sf_n) graph and umbrella graph is obtained ($U_{m,n}$) as follows.

Theorem 1 For every integer the $n \geq 4$, local edge (a, d) –antimagic coloring of the graph $(Sf_n)$is $\chi_{le(3,1)}(Sf_n) = 3n$.

Theorem 2 For every integer the $n \geq 3$ and $m \geq 3$, local edge (a, d) –antimagic coloring of an umbrella graph $(U_{m,n})$ is $\chi_{le\left(\frac{3n}{2},1\right)}(U_{m,n}) = m + 1$.

Based on the theorem of local edge (a, d) –antimagic coloring in the sunflower and umbrella graphs, we developed the new Sidoarjo batik motif design.

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